

# PCoMS OpenMX Hands-on Tutorial

## Program:

9:30-10:40	Introduction of DFT and OpenMX
10:40-11:10	Hands-on lecture
11:10-12:00	Login to computer and perform test calculations
12:00-13:00	Lunch
13:00-13:30	Geometry optimization, NEB calculations, and molecular dynamics
13:30-14:10	Practical session (take a break properly)
14:10-14:40	Calculations of X-ray photoemission spectra
14:40-15:20	Practical session (take a break properly)
15:20-15:50	Electronic transport calculations by NEGF
15:50-16:30	Practical session (take a break properly)
16:30-17:00	Discussion session

Taisuke Ozaki (ISSP, Univ. of Tokyo)

Mitsuaki Kawamura (ISSP, Univ. of Tokyo)

Institute for Materials Research (IMR), Tohoku University, Feb. 14, 2020.

# Purposes of the tutorial

The first-principles calculations based on density functional theories (DFT) have been playing an indispensable role in deeply understanding physical and chemical properties of materials in recent years.

In the tutorial, we would like to introduce OpenMX based on DFT, optimized numerical basis orbitals, and pseudopotential with practical calculations so that that you can get familiar with OpenMX calculations.

Any questions regarding OpenMX calculations are welcome. We hope that OpenMX can be useful for your future researches.

# OpenMX **Open** source package for **M**aterial **eX**plorer

- Software package for density functional calculations of molecules and bulks
- Norm-conserving pseudopotentials (PPs)
- Variationally optimized numerical atomic basis functions

## Basic functionalities

- SCF calc. by LDA, GGA, DFT+U
- Total energy and forces on atoms
- Band dispersion and density of states
- Geometry optimization by BFGS, RF, EF
- Charge analysis by Mulliken, Voronoi, ESP
- Molecular dynamics with NEV and NVT ensembles
- Charge doping
- Fermi surface
- Analysis of charge, spin, potentials by cube files
- Database of optimized PPs and basis functions

## Extensions

- O(N) and low-order scaling diagonalization
- Non-collinear DFT for non-collinear magnetism
- Spin-orbit coupling included self-consistently
- Electronic transport by non-equilibrium Green function
- Electronic polarization by the Berry phase formalism
- Maximally localized Wannier functions
- Effective screening medium method for biased system
- Reaction path search by the NEB method
- Band unfolding method
- STM image by the Tersoff-Hamann method
- etc.

# History of OpenMX

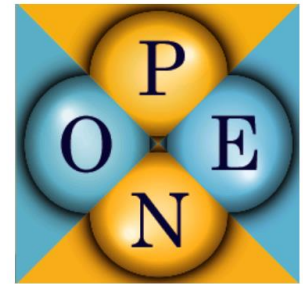
- 2000 Start of development
- 2003 Public release (GNU-GPL)
- 2003 Collaboration:  
AIST, NIMS, SNU  
KAIST, JAIST,  
Kanazawa Univ.  
CAS, UAM  
NISSAN, Fujitsu Labs.  
etc.
- 2019 19 public releases  
Latest version: 3.9

Welcome to OpenMX  
open source package for Material eXplorer

Google C

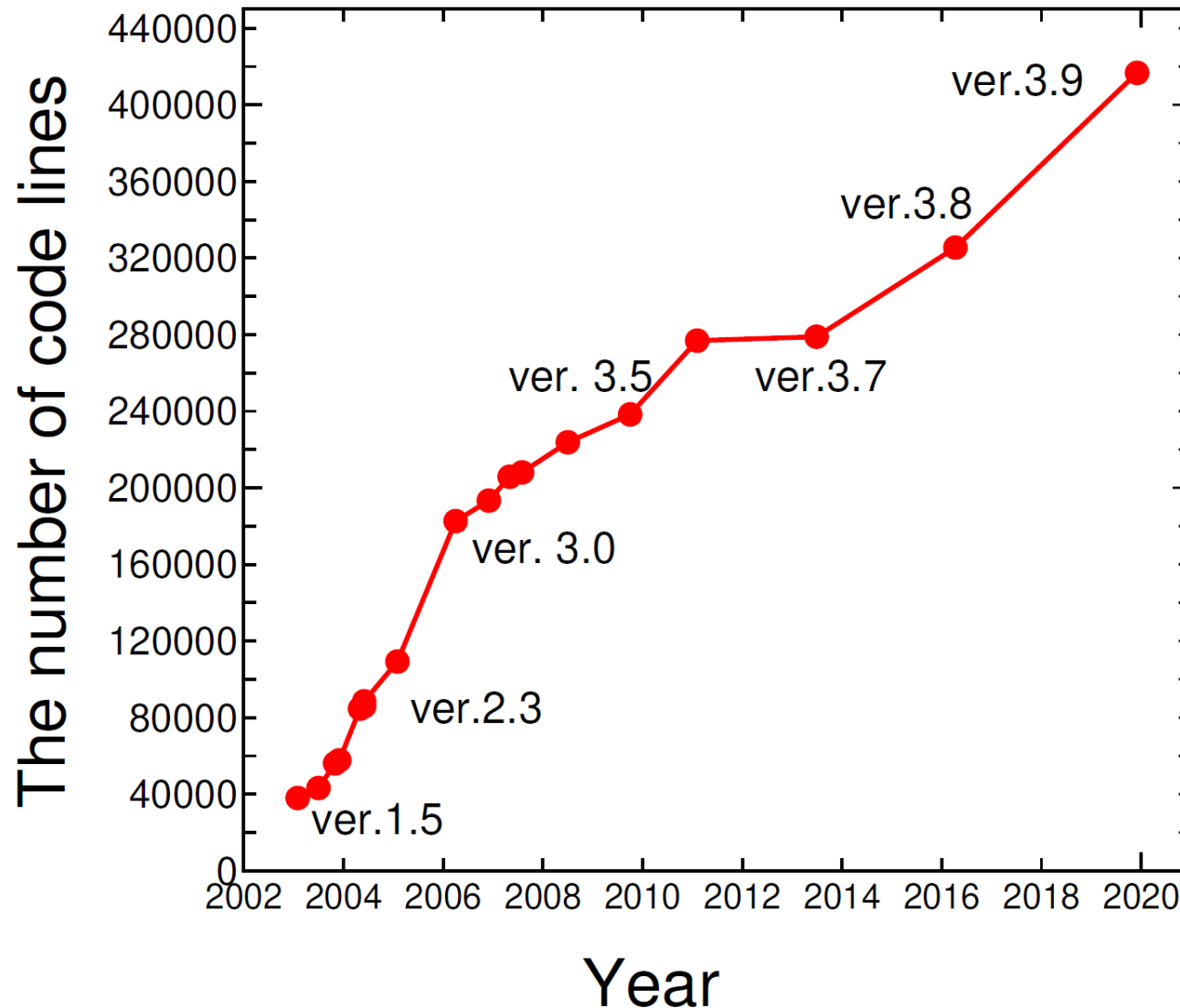
## Contents

- **What's new**
  - Patch (Ver. 3.8.5) to OpenMX Ver. 3.8 (June 12, 2018)
- **What is OpenMX?**
- **Download**
- **Manual of Ver. 3.8**
- **Technical Notes**
- **Video Lectures**
- **Publications**
- **OpenMX Forum**
- **OpenMX Viewer**
- **Workshop**
- **Database of Results**
- **Database of VPS and PAO**
  - Ver. 2019
  - Ver. 2019 for core excitations
- **ADPACK**
- **Miscellaneous informations**
- **Contributors**
- **Acknowledgment**
- **Opening positions**
- **Links**



<http://www.openmx-square.org>

# Development of OpenMX code



# Contributors to OpenMX development

T. Ozaki (Univ. of Tokyo)  
H. Kino (NIMS)  
J. Yu (SNU)  
M.J. Han (KAIST)  
M. Ohfuchi (Fujitsu Labs.)  
F. Ishii (Kanazawa Univ.)  
K. Sawada (Kanazawa Univ.)  
Y. Kubota (Kanazawa Univ.)  
Y.P. Mizuta (Kanazawa Univ.)  
H. Kotaka (Kyoto Univ.)  
N. Yamaguchi (Kanazawa Univ.)  
H. Sawahata (Kanazawa Univ.)  
T.B. Prayitno (Kanazawa Univ.)  
T. Ohwaki (NISSAN ARC)  
T.V.T Duy (AISIN SEIKI)  
M. Miyata (JAIST)  
G. Jiang (Wuhan Univ. of Sci.&Tech.)  
T. Iitaka (RIKEN)

P.-H. Chang (George Mason Univ.)  
A. Terasawa (TIT)  
Y. Gohda (TIT)  
H. Weng (CAS)  
Y. Shiihara (Toyota Tech. Inst.)  
M. Toyoda (Tokyo Inst. Tech.)  
Y. Okuno (FUJIFILM)  
R. Perez (UAM)  
P.P. Bell (UAM)  
M. Ellner (UAM)  
Yang Xiao (NUAA)  
A.M. Ito (NIFS)  
M. Otani (AIST)  
M. Kawamura (Univ. of Tokyo)  
K. Yoshimi (Univ. of Tokyo)  
C.-C. Lee (Tamkang Univ.)  
Y.-T. Lee (Academia Sinica)  
M. Fukuda (Univ. of Tokyo)  
S. Ryee (KAIST)  
K. Terakura (AIST)

# Materials studied by OpenMX

## First characterization of silicene on $\text{ZrB}_2$ in collaboration with experimental groups

A. Fleurence et al., Phys. Rev. Lett. 108, 245501 (2012).

## First identification of $J_{\text{eff}}=1/2$ Mott state of Ir oxides

B.J. Kim et al., Phys. Rev. Lett. 101, 076402 (2008).

## Theoretical proposal of topological insulators

C.-H. Kim et al., Phys. Rev. Lett. 108, 106401 (2012).

H. Weng et al., Phys. Rev. X 4, 011002 (2014).

## First-principles molecular dynamics simulations for Li ion battery

T. Ohwaki et al., J. Chem. Phys. 136, 134101 (2012).

T. Ohwaki et al., J. Chem. Phys. 140, 244105 (2014).

## Magnetic anisotropy energy of magnets

Z. Torbatian et al., Appl. Phys. Lett. 104, 242403 (2014).

I. Kitagawa et al., Phys. Rev. B 81, 214408 (2010).

## Electronic transport of graphene nanoribbon on surface oxidized Si

H. Jippo et al., Appl. Phys. Express 7, 025101 (2014).

M. Ohfuchi et al., Appl. Phys. Express 4, 095101 (2011).

## Interface structures of carbide precipitate in bcc-Fe

H. Sawada et al., Modelling Simul. Mater. Sci. Eng. 21, 045012 (2013).

## Universality of medium range ordered structure in amorphous metal oxides

K. Nishio et al., Phys. Rev. Lett. 340, 155502 (2013).

## Materials treated so far

Silicene, graphene

Carbon nanotubes

Transition metal oxides

Topological insulators

Intermetallic compounds

Molecular magnets

Rare earth magnets

Lithium ion related materials

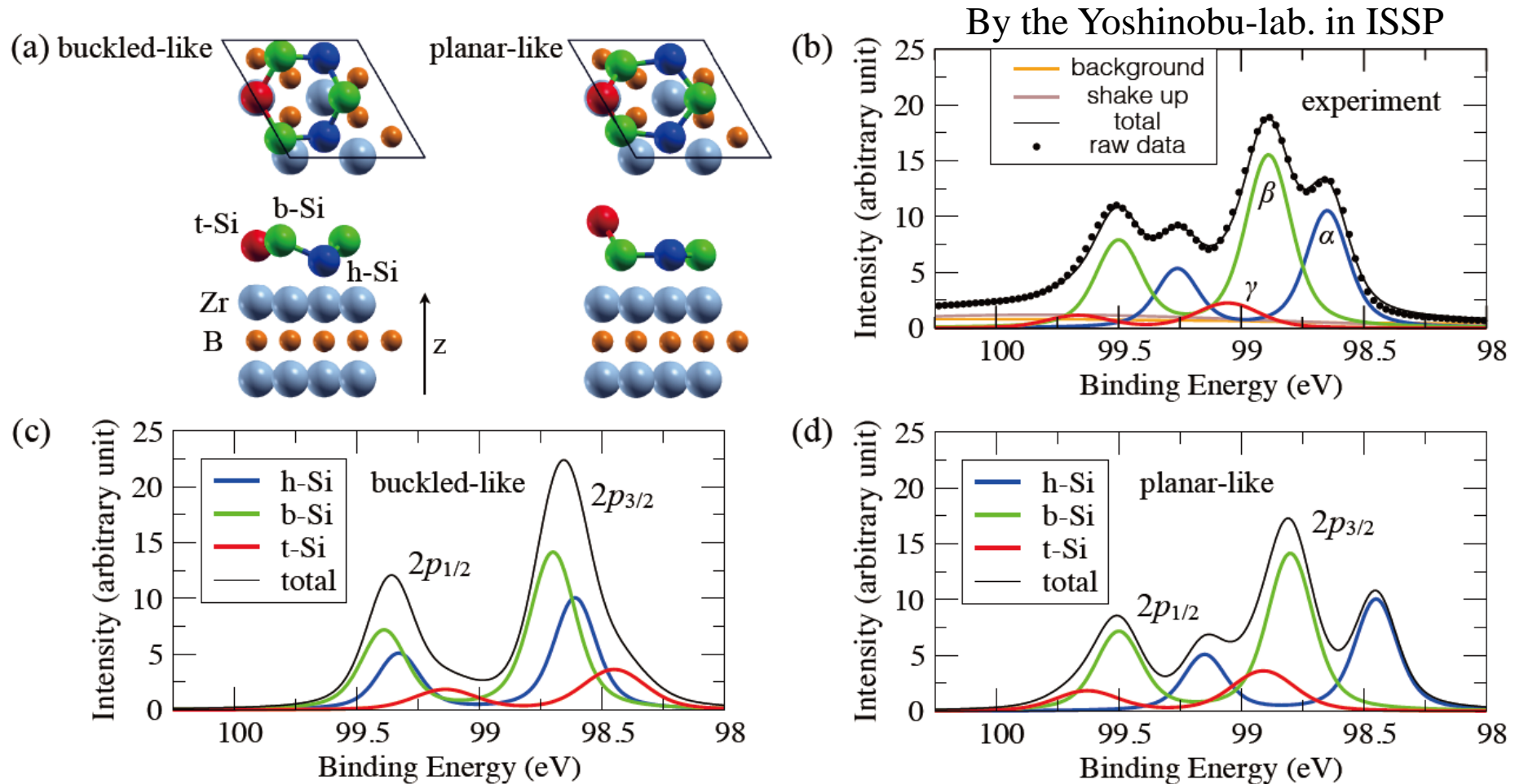
Structural materials

etc.

**About 800 published papers**

# Silicene on $\text{ZrB}_2$

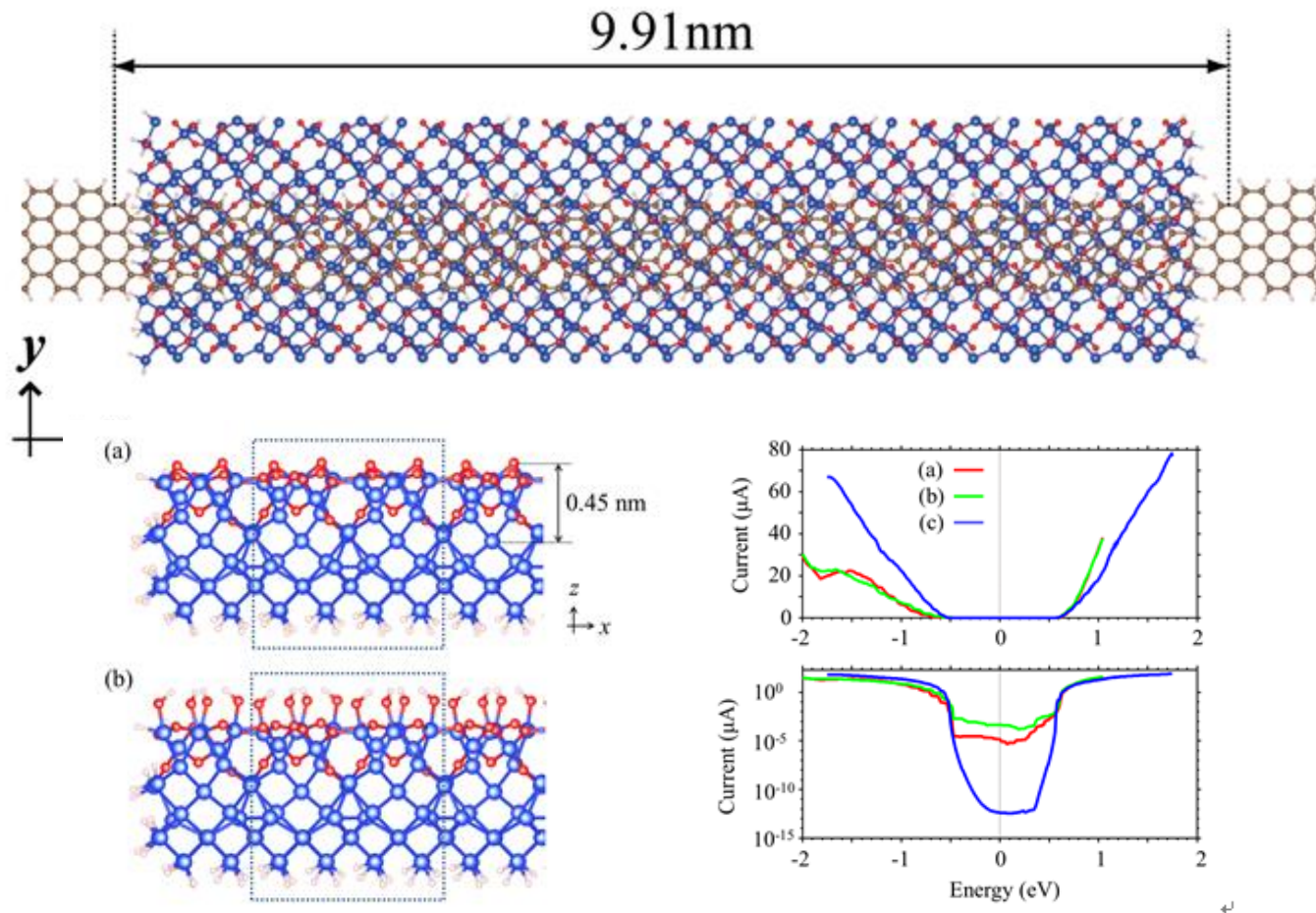
The XPS data is well compared with the calculated binding energy of planar-like structure.





# Electron transport of graphene nanoribbon

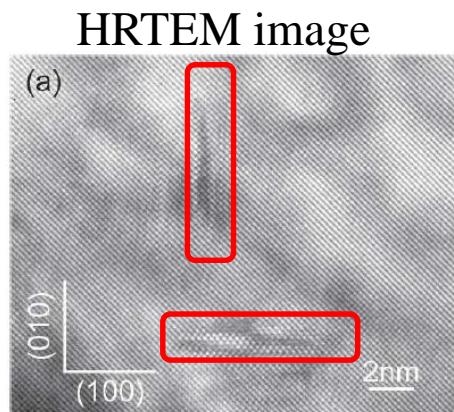
The NEGF calculations predict that the on/off ratio of conductance of the graphene nanoribbon on surface oxidized Si substrates is about  $10^5$ , being consistent with experimental measurements.



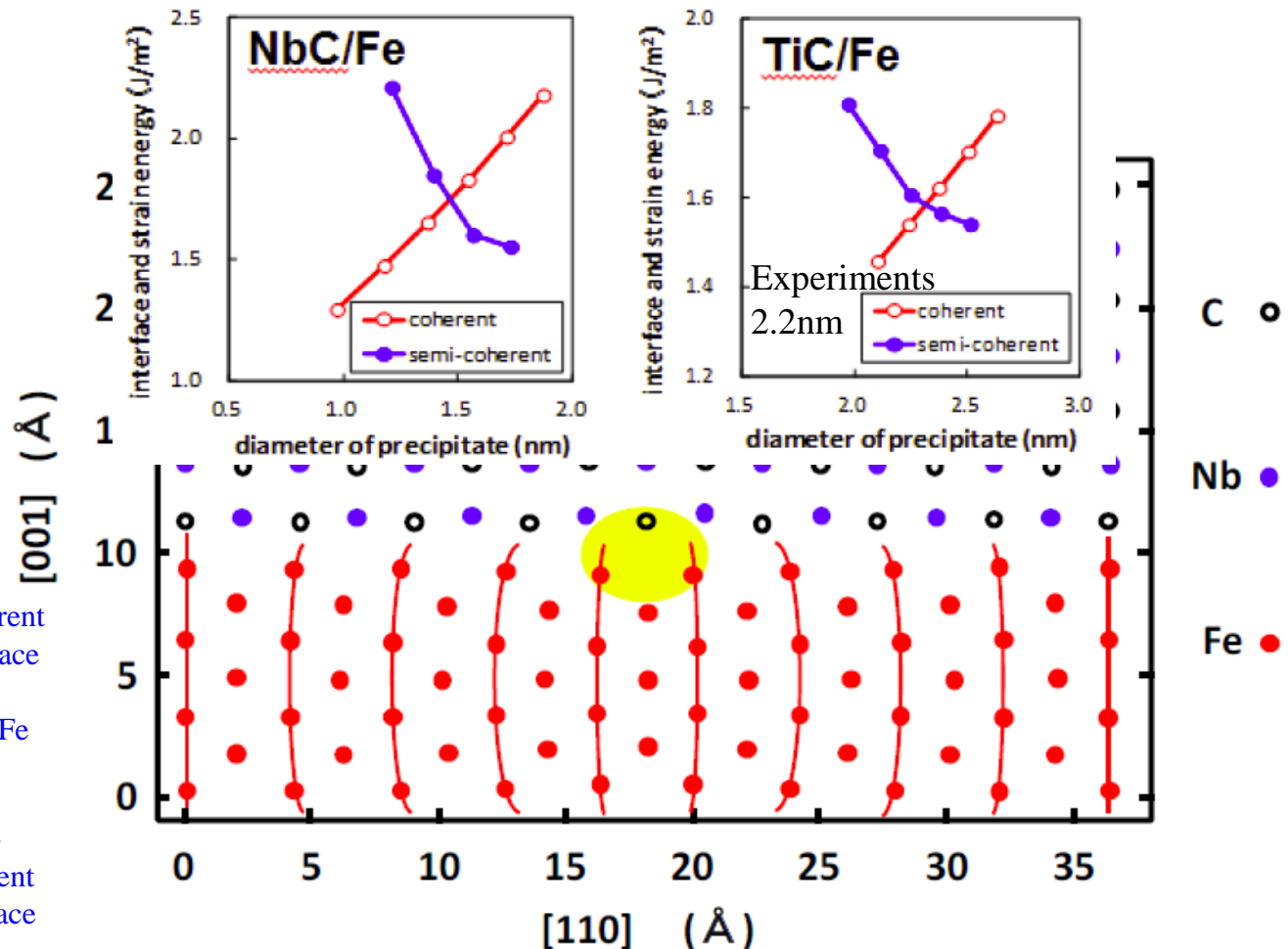
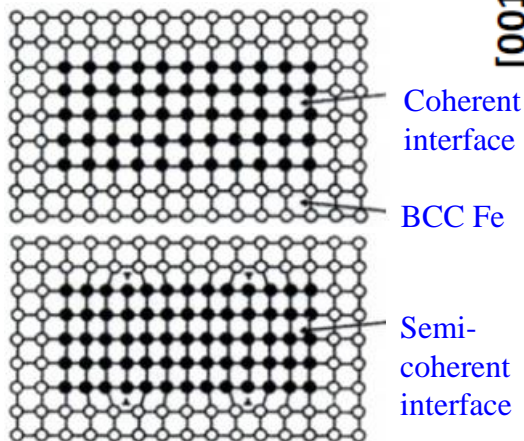
# Interface of BCC-iron and metal carbides

The precipitation of metal carbides is an effective way to control the hardness and toughness of steel. We tried to estimate possible interface structures depending on the size of precipitates.

H. Sawada et al., Modelling Simul. Mater. Sci. Eng. 21, 045012 (2013).



Precipitates: TiC, VC, NbC



# Implementation of OpenMX

- Density functional theory
- Mathematical structure of KS eq.
- LCPAO method
- Total energy
- Pseudopotentials
- Basis functions

# Density functional theory

The energy of non-degenerate ground state can be expressed by a functional of electron density. (Hohenberg and Kohn, 1964)

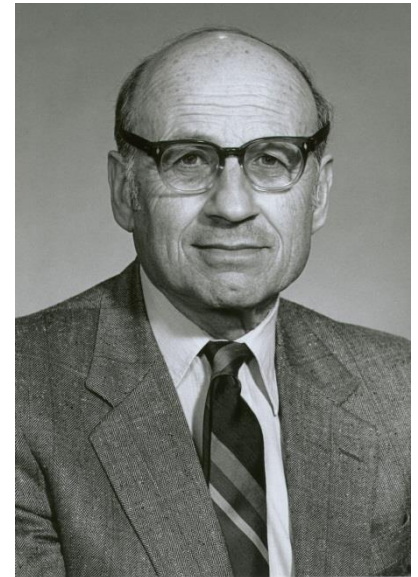
$$E[\rho] = \int \rho(\mathbf{r})v(\mathbf{r})d + T[\rho] + J[\rho] + E_{\text{xc}}[\rho]$$

The many body problem of the ground state can be reduced to an one-particle problem with an effective potential. (Kohn-Sham, 1965)

$$\hat{H}_{\text{KS}}\phi_i = \varepsilon_i\phi_i$$

$$\hat{H}_{\text{KS}} = -\frac{1}{2}\nabla^2 + v_{\text{eff}}$$

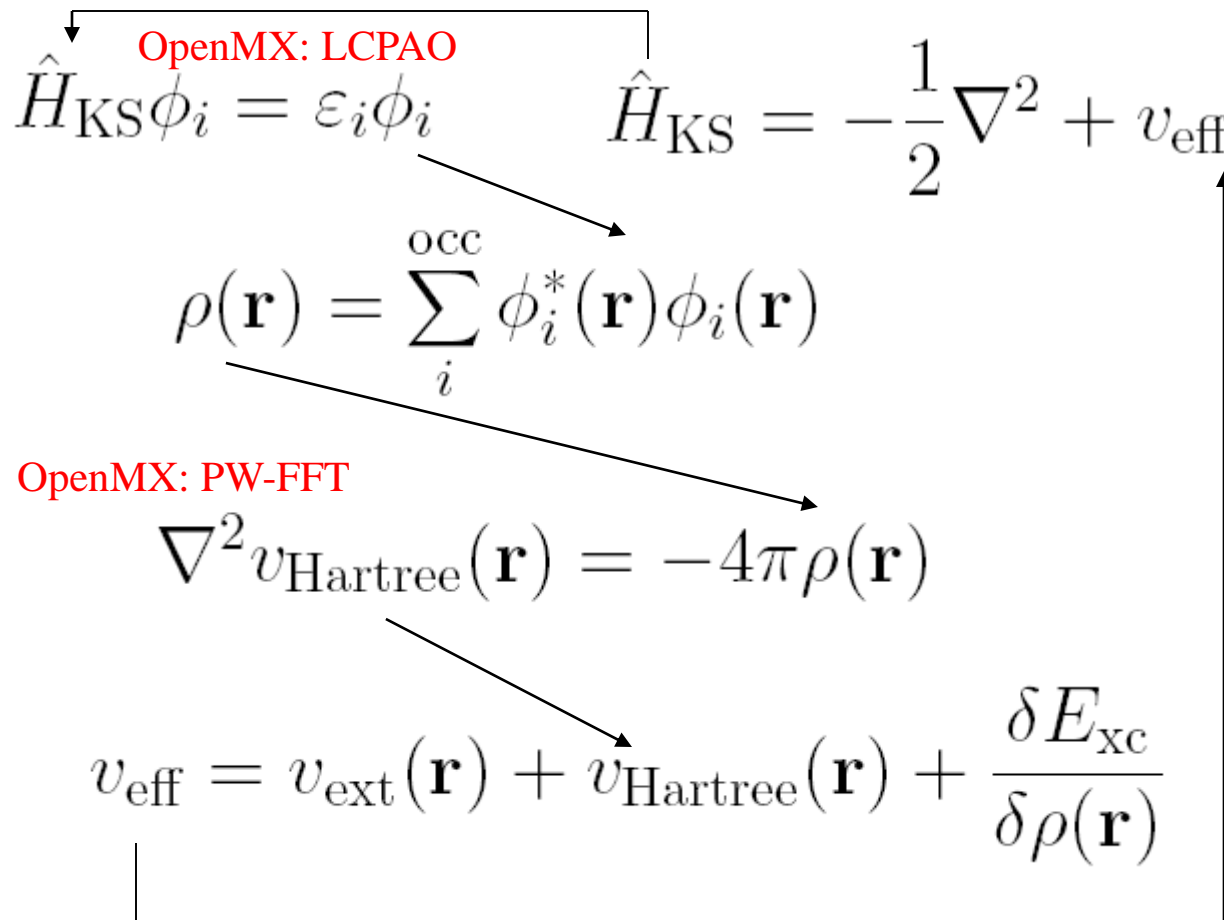
$$v_{\text{eff}} = v_{\text{ext}}(\mathbf{r}) + v_{\text{Hartree}}(\mathbf{r}) + \frac{\delta E_{\text{xc}}}{\delta \rho(\mathbf{r})}$$



W.Kohn (1923-2016)

# Algorithmic structure of KS eq.

3D coupled non-linear differential equations have to be solved self-consistently.



Input charge = Output charge  $\rightarrow$  Self-consistent condition

# LCPAO method

## (Linear-Combination of Pseudo Atomic Orbital Method)

One-particle KS orbital

$$\psi_{\sigma\mu}^{(\mathbf{k})}(\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{\mathbf{n}} e^{i\mathbf{R}_{\mathbf{n}} \cdot \mathbf{k}} \sum_{i\alpha} c_{\sigma\mu,i\alpha}^{(\mathbf{k})} \phi_{i\alpha}(\mathbf{r} - \tau_i - \mathbf{R}_{\mathbf{n}}),$$

is expressed by a linear combination of atomic like orbitals in the method.

$$\phi(\mathbf{r}) = Y_l^m(\hat{\mathbf{r}}) R(r)$$

### Features:

- It is easy to interpret physical and chemical meanings, since the KS orbitals are expressed by the atomic like basis functions.
- It gives rapid convergent results with respect to basis functions due to physical origin. (however, it is not a complete basis set, leading to difficulty in getting full convergence.)
- The memory and computational effort for calculation of matrix elements are  $O(N)$ .
- It well matches the idea of linear scaling methods.

➤ **Total energy**

➤ Pseudopotentials

➤ Basis functions

# Implementation: Total energy (1)

The total energy is given by the sum of six terms, and a proper integration scheme for each term is applied to accurately evaluate the total energy.

$$E_{\text{tot}} = E_{\text{kin}} + E_{\text{ec}} + E_{\text{ee}} + E_{\text{xc}} + E_{\text{cc}} = E_{\text{kin}} + E_{\text{na}} + E_{\text{ec}}^{(\text{NL})} + E_{\delta\text{ee}} + E_{\text{xc}} + E_{\text{scc}}.$$

$$E_{\text{kin}} = \sum_{\sigma} \sum_{\mathbf{n}} \sum_{i\alpha, j\beta} \rho_{\sigma, i\alpha j\beta}^{(\mathbf{R}_{\mathbf{n}})} h_{i\alpha j\beta, \text{kin}}^{(\mathbf{R}_{\mathbf{n}})}. \quad \text{Kinetic energy}$$

$$\begin{aligned} E_{\text{ec}} &= E_{\text{ec}}^{(\text{L})} + E_{\text{ec}}^{(\text{NL})}, \quad \text{Coulomb energy with external potential} \\ &= \sum_{\sigma} \sum_{\mathbf{n}} \sum_{i\alpha, j\beta} \rho_{\sigma, i\alpha j\beta}^{(\mathbf{R}_{\mathbf{n}})} \langle \phi_{i\alpha}(\mathbf{r} - \tau_i) | \sum_I V_{\text{core}, I}(\mathbf{r} - \tau_I) | \phi_{j\beta}(\mathbf{r} - \tau_j - \mathbf{R}_{\mathbf{n}}) \rangle \\ &\quad + \sum_{\sigma} \sum_{\mathbf{n}} \sum_{i\alpha, j\beta} \rho_{\sigma, i\alpha j\beta}^{(\mathbf{R}_{\mathbf{n}})} \langle \phi_{i\alpha}(\mathbf{r} - \tau_i) | \sum_I V_{\text{NL}, I}(\mathbf{r} - \tau_I) | \phi_{j\beta}(\mathbf{r} - \tau_j - \mathbf{R}_{\mathbf{n}}) \rangle, \end{aligned}$$

$$\begin{aligned} E_{\text{ee}} &= \frac{1}{2} \int d\mathbf{r}^3 n(\mathbf{r}) V_{\text{H}}(\mathbf{r}), \quad \text{Hartree energy} \\ &= \frac{1}{2} \int d\mathbf{r}^3 n(\mathbf{r}) \{ V_{\text{H}}^{(\text{a})}(\mathbf{r}) + \delta V_{\text{H}}(\mathbf{r}) \}, \end{aligned}$$

$$E_{\text{xc}} = \int d\mathbf{r}^3 \{ n_{\uparrow}(\mathbf{r}) + n_{\downarrow}(\mathbf{r}) + n_{\text{pcc}}(\mathbf{r}) \} \epsilon_{\text{xc}}(n_{\uparrow} + \frac{1}{2}n_{\text{pcc}}, n_{\downarrow} + \frac{1}{2}n_{\text{pcc}}), \quad \text{Exchange-correlation energy}$$

$$E_{\text{cc}} = \frac{1}{2} \sum_{I, J} \frac{Z_I Z_J}{|\tau_I - \tau_J|}. \quad \text{Core-core Coulomb energy}$$



# Implementation: Total energy (2)

The reorganization of Coulomb energies gives three new energy terms.

$$E_{\text{ec}}^{(L)} + E_{\text{ee}} + E_{\text{cc}} = E_{\text{na}} + E_{\delta\text{ee}} + E_{\text{scc}},$$

The neutral atom energy

$$E_{\text{na}} = \int d\mathbf{r}^3 n(\mathbf{r}) \sum_I V_{\text{na},I}(\mathbf{r} - \tau_I),$$

Short range and separable to two-center integrals

$$= \sum_{\sigma} \sum_{\mathbf{n}} \sum_{i\alpha, j\beta} \rho_{\sigma, i\alpha j\beta}^{(\mathbf{R}_{\mathbf{n}})} \sum_I \langle \phi_{i\alpha}(\mathbf{r} - \tau_i) | V_{\text{na},I}(\mathbf{r} - \tau_I) | \phi_{j\beta}(\mathbf{r} - \tau_j - \mathbf{R}_{\mathbf{n}}) \rangle,$$

Difference charge Hartree energy

$$E_{\delta\text{ee}} = \frac{1}{2} \int d\mathbf{r}^3 \delta n(\mathbf{r}) \delta V_{\text{H}}(\mathbf{r}),$$

Long range but minor contribution

Screened core-core repulsion energy

$$E_{\text{scc}} = \frac{1}{2} \sum_{I,J} \left[ \frac{Z_I Z_J}{|\tau_I - \tau_J|} - \int d\mathbf{r}^3 n_I^{(\text{a})}(\mathbf{r}) V_{\text{H},J}^{(\text{a})}(\mathbf{r}) \right].$$

Short range and two-center integrals

Difference charge

$$\begin{aligned} \delta n(\mathbf{r}) &= n(\mathbf{r}) - n^{(\text{a})}(\mathbf{r}), \\ &= n(\mathbf{r}) - \sum_i n_i^{(\text{a})}(\mathbf{r}), \end{aligned}$$

Neutral atom potential

$$V_{\text{na},I}(\mathbf{r} - \tau_I) = V_{\text{core},I}(\mathbf{r} - \tau_I) + V_{\text{H},I}^{(\text{a})}(\mathbf{r} - \tau_I).$$

# Implementation: Total energy (3)

So, the total energy is given by

$$E_{\text{tot}} = E_{\text{kin}} + E_{\text{na}} + E_{\text{ec}}^{(\text{NL})} + E_{\delta\text{ee}} + E_{\text{xc}} + E_{\text{scc}}.$$

Each term is evaluated by using a different numerical grid with consideration on accuracy and efficiency.

$$\left. \begin{array}{l} E_{\text{kin}} \\ E_{\text{na}} \\ E_{\text{ec}}^{(\text{NL})} \end{array} \right\} \text{ Spherical coordinate in momentum space}$$

$$\left. \begin{array}{l} E_{\delta\text{ee}} \\ E_{\text{xc}} \end{array} \right\} \text{ Real space regular mesh}$$

$$E_{\text{scc}} \quad \text{Real space fine mesh}$$

# Two center integrals

## Fourier-transformation of basis functions

$$\begin{aligned}
 \tilde{\phi}_{i\alpha}(\mathbf{k}) &= \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int d\mathbf{r}^3 \phi_{i\alpha}(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}} \\
 &= \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int d\mathbf{r}^3 Y_{lm}(\hat{\mathbf{r}}) R_{pl}(r) \left\{ 4\pi \sum_{L=0}^{\infty} \sum_{M=-L}^L (-i)^L j_L(kr) Y_{LM}(\hat{\mathbf{k}}) Y_{LM}^*(\hat{\mathbf{r}}) \right\}, \\
 &= \left(\frac{1}{\sqrt{2\pi}}\right)^3 4\pi \sum_{L=0}^{\infty} \sum_{M=-L}^L (-i)^L Y_{LM}(\hat{\mathbf{k}}) \int dr r^2 R_{pl}(r) j_L(kr) \int d\theta d\phi \sin(\theta) Y_{lm}(\hat{\mathbf{r}}) Y_{LM}^*(\hat{\mathbf{r}}), \\
 &= \left[ \left(\frac{1}{\sqrt{2\pi}}\right)^3 4\pi (-i)^l \int dr r^2 R_{pl}(r) j_L(kr) \right] Y_{lm}(\hat{\mathbf{k}}), \\
 &= \tilde{R}_{pl}(k) Y_{lm}(\hat{\mathbf{k}}),
 \end{aligned}$$

Integrals for angular parts are analytically performed. Thus, we only have to perform one-dimensional integrals along the radial direction.

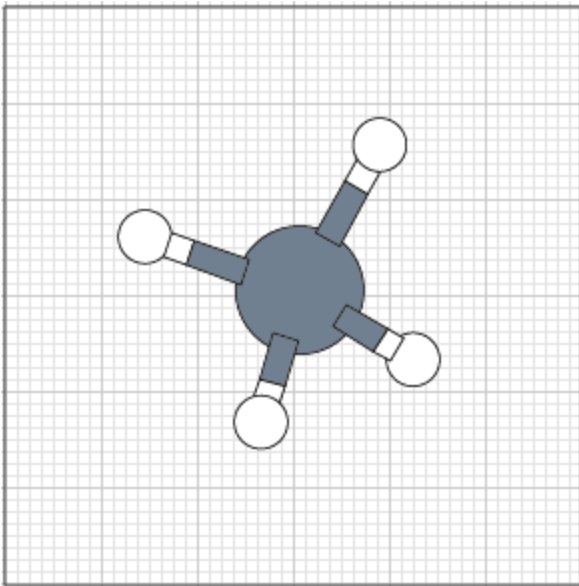
## e.g., overlap integral

$$\begin{aligned}
 \langle \phi_{i\alpha}(\mathbf{r}) | \phi_{j\beta}(\mathbf{r} - \tau) \rangle &= \int d\mathbf{r}^3 \phi_{i\alpha}^*(\mathbf{r}) \phi_{j\beta}(\mathbf{r} - \tau), \\
 &= \int d\mathbf{r}^3 \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int dk^3 \tilde{R}_{pl}^*(k) Y_{lm}^*(\hat{\mathbf{k}}) e^{-i\mathbf{k}\cdot\mathbf{r}} \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int dk'^3 \tilde{R}_{p'l'}(k') Y_{l'm'}(\hat{\mathbf{k}}') e^{i\mathbf{k}'\cdot(\mathbf{r}-\tau)}, \\
 &= \left(\frac{1}{2\pi}\right)^3 \int dk^3 \int dk'^3 e^{-i\mathbf{k}'\cdot\tau} \tilde{R}_{pl}^*(k) Y_{lm}^*(\hat{\mathbf{k}}) \tilde{R}_{p'l'}(k') Y_{l'm'}(\hat{\mathbf{k}}') \int d\mathbf{r}^3 e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}}, \\
 &= \int dk^3 e^{-i\mathbf{k}\cdot\tau} \tilde{R}_{pl}^*(k) Y_{lm}^*(\hat{\mathbf{k}}) \tilde{R}_{p'l'}(k) Y_{l'm'}(\hat{\mathbf{k}}),
 \end{aligned}$$

# Cutoff energy for regular mesh

The two energy components  $E_{\delta ee} + E_{xc}$  are calculated on real space regular mesh. The mesh fineness is determined by plane-wave cutoff energies.

```
scf.energycutoff      150.0      # default=150 (Ry)
```



The cutoff energy can be related to the mesh fineness by the following eqs.

$$E_{\text{cut}}^{(1)} = \frac{1}{2} \mathbf{g}\mathbf{b}_1 \cdot \mathbf{g}\mathbf{b}_1, \quad E_{\text{cut}}^{(2)} = \frac{1}{2} \mathbf{g}\mathbf{b}_2 \cdot \mathbf{g}\mathbf{b}_2, \quad E_{\text{cut}}^{(3)} = \frac{1}{2} \mathbf{g}\mathbf{b}_3 \cdot \mathbf{g}\mathbf{b}_3,$$

$$\mathbf{g}\mathbf{a}_1 = \frac{\mathbf{a}_1}{N_1}, \quad \mathbf{g}\mathbf{a}_2 = \frac{\mathbf{a}_2}{N_2}, \quad \mathbf{g}\mathbf{a}_3 = \frac{\mathbf{a}_3}{N_3},$$

$$\mathbf{g}\mathbf{b}_1 = 2\pi \frac{\mathbf{g}\mathbf{a}_2 \times \mathbf{g}\mathbf{a}_3}{\Delta V}, \quad \mathbf{g}\mathbf{b}_2 = 2\pi \frac{\mathbf{g}\mathbf{a}_3 \times \mathbf{g}\mathbf{a}_1}{\Delta V}, \quad \mathbf{g}\mathbf{b}_3 = 2\pi \frac{\mathbf{g}\mathbf{a}_1 \times \mathbf{g}\mathbf{a}_2}{\Delta V},$$

$$\Delta V = \mathbf{g}\mathbf{a}_1 \cdot (\mathbf{g}\mathbf{a}_2 \times \mathbf{g}\mathbf{a}_3),$$

# Forces

$$\begin{aligned} \mathbf{F}_i &= -\frac{\partial E_{\text{tot}}}{\partial \mathbf{R}_i} \\ &= -\boxed{\frac{\partial E_{\text{kin}}}{\partial \mathbf{R}_i}} - \boxed{\frac{\partial E_{\text{na}}}{\partial \mathbf{R}_i}} - \boxed{\frac{\partial E_{\text{scc}}}{\partial \mathbf{R}_i}} - \boxed{\frac{\partial E_{\delta\text{ee}}}{\partial \mathbf{R}_i}} - \boxed{\frac{\partial E_{\text{xc}}}{\partial \mathbf{R}_i}} - \boxed{\frac{\partial E_{\text{cc}}}{\partial \mathbf{R}_i}} \end{aligned}$$

$$\frac{\partial E_{\delta\text{ee}}}{\partial \mathbf{R}_k} = \sum_{\mathbf{p}} \frac{\partial n(\mathbf{r}_{\mathbf{p}})}{\partial \mathbf{R}_k} \frac{\partial E_{\delta\text{ee}}}{\partial n(\mathbf{r}_{\mathbf{p}})} + \sum_{\mathbf{p}} \frac{\partial n^{\text{a}}(\mathbf{r}_{\mathbf{p}})}{\partial \mathbf{R}_k} \frac{\partial E_{\delta\text{ee}}}{\partial n^{\text{a}}(\mathbf{r}_{\mathbf{p}})}.$$

$$\begin{aligned} \frac{\partial E_{\delta\text{ee}}}{\partial n(\mathbf{r}_{\mathbf{p}})} &= \frac{1}{2} \Delta V \{ \delta V_{\text{H}}(\mathbf{r}_{\mathbf{p}}) + \sum_{\mathbf{q}} \delta n(\mathbf{r}_{\mathbf{q}}) \frac{\partial \delta V_{\text{H}}(\mathbf{r}_{\mathbf{q}})}{\partial n(\mathbf{r}_{\mathbf{p}})} \}, \\ &= \frac{1}{2} \Delta V \{ \delta V_{\text{H}}(\mathbf{r}_{\mathbf{p}}) + \frac{4\pi}{N_{\text{rsg}}} \sum_{\mathbf{G}} \frac{1}{|\mathbf{G}|^2} \sum_{\mathbf{q}} \delta n(\mathbf{r}_{\mathbf{q}}) e^{i\mathbf{G} \cdot (\mathbf{r}_{\mathbf{q}} - \mathbf{r}_{\mathbf{p}})} \}, \\ &= \Delta V \delta V_{\text{H}}(\mathbf{r}_{\mathbf{p}}). \end{aligned}$$



Easy calc.



See the left

$$\begin{aligned} \frac{\partial E_{\delta\text{ee}}}{\partial n^{\text{a}}(\mathbf{r}_{\mathbf{p}})} &= -\frac{1}{2} \Delta V \{ \delta V_{\text{H}}(\mathbf{r}_{\mathbf{p}}) - \sum_{\mathbf{q}} \delta n(\mathbf{r}_{\mathbf{q}}) \frac{\partial \delta V_{\text{H}}(\mathbf{r}_{\mathbf{q}})}{\partial n^{\text{a}}(\mathbf{r}_{\mathbf{p}})} \}, \\ &= -\frac{1}{2} \Delta V \{ \delta V_{\text{H}}(\mathbf{r}_{\mathbf{p}}) + \frac{4\pi}{N_{\text{rsg}}} \sum_{\mathbf{G}} \frac{1}{|\mathbf{G}|^2} \sum_{\mathbf{q}} \delta n(\mathbf{r}_{\mathbf{q}}) e^{i\mathbf{G} \cdot (\mathbf{r}_{\mathbf{q}} - \mathbf{r}_{\mathbf{p}})} \}, \\ &= -\Delta V \delta V_{\text{H}}(\mathbf{r}_{\mathbf{p}}). \end{aligned}$$

$$\begin{aligned} \frac{\partial n(\mathbf{r}_{\mathbf{p}})}{\partial \mathbf{R}_k} &= \sum_{i\alpha,j\beta} \sum_{\nu} \{ \frac{\partial c_{i\alpha,\nu}^*}{\partial \mathbf{R}_k} c_{j\beta,\nu} \chi_{i\alpha}(\mathbf{r}) \chi_{j\beta}(\mathbf{r}) + c_{i\alpha,\nu}^* \frac{\partial c_{j\beta,\nu}}{\partial \mathbf{R}_k} \chi_{i\alpha}(\mathbf{r}_{\mathbf{p}}) \chi_{j\beta}(\mathbf{r}_{\mathbf{p}}) \} \\ &\quad + 2 \sum_{\alpha,j\beta} \rho_{k\alpha,j\beta} \frac{\partial \chi_{k\alpha}(\mathbf{r}_{\mathbf{p}})}{\partial \mathbf{R}_k} \chi_{j\beta}(\mathbf{r}_{\mathbf{p}}). \end{aligned}$$

$$\begin{aligned} \frac{\partial E_{\text{xc}}}{\partial \mathbf{R}_k} &= \sum_{\mathbf{p}} \frac{\partial n(\mathbf{r}_{\mathbf{p}})}{\partial \mathbf{R}_k} \frac{\partial E_{\text{xc}}}{\partial n(\mathbf{r}_{\mathbf{p}})}, \\ &= \Delta V \sum_{\mathbf{p}} \frac{\partial n(\mathbf{r}_{\mathbf{p}})}{\partial \mathbf{R}_k} v_{\text{xc}}(n(\mathbf{r}_{\mathbf{p}})). \end{aligned}$$

Forces are always analytic at any grid fineness and at zero temperature, even if numerical basis functions and numerical grids.

- Total energy
- **Pseudopotentials**
- Basis functions

# Norm-conserving Vanderbilt pseudopotential

I. Morrion, D.M. Bylander, and L. Kleinman, PRB 47, 6728 (1993).

The following non-local operator proposed by Vanderbilt guarantees that scattering properties are reproduced **around multiple reference energies**.

D. Vanderbilt, PRB 41, 7892 (1990).

$$V_{\text{NL}} = \sum_{i,j} B_{ij} |\beta_i\rangle \langle \beta_j|$$

$$|\chi_i\rangle = V_{\text{NL}}^{(i)} |\phi_i\rangle = (\varepsilon_i - T - V_{\text{loc}}) |\phi_i\rangle$$

$$B_{ij} = \langle \phi_i | \chi_j \rangle$$

$$|\beta_i\rangle = \sum_j (B^{-1})_{ji} |\chi_j\rangle$$

If the following generalized norm-conserving condition is fulfilled, the matrix B is Hermitian, resulting in that  $V_{\text{NL}}$  is also Hermitian.

$$Q_{ij} = \langle \psi_i | \psi_j \rangle_R - \langle \phi_i | \phi_j \rangle_R$$

If  $Q=0$ , then  $B-B^*=0$

$$B_{ij} - B_{ji}^* = (\varepsilon_i - \varepsilon_j) Q_{ij}$$

This is the norm-conserving PP  
used in OpenMX

# Non-local potentials by Vanderbilt

Let's operate the non-local potential on a pseudized wave function:

$$\begin{aligned}
 \hat{v}^{(\text{NL})} |\phi_k^{(\text{PS})}\rangle &= \sum_{ij} |\beta_i\rangle B_{ij} \langle \beta_j | \phi_k^{(\text{PS})} \rangle \\
 &= \sum_{ij} |\beta_i\rangle B_{ij} \sum_{k'} (B^{-1})_{k'j} \langle \chi_{k'} | \phi_k^{(\text{PS})} \rangle, \quad \text{Noting the following relations:} \\
 &= \sum_{ij} |\beta_i\rangle B_{ij} \sum_{k'} (B^{-1})_{k'j} B_{kk'}, \\
 &= \sum_{ij} |\beta_i\rangle B_{ij} \delta_{kj}, \\
 &= \sum_i \left( \sum_j (B^{-1})_{ji} |\chi_j\rangle \right) B_{ik}. \\
 &= \boxed{|\chi_k\rangle}
 \end{aligned}$$

$$v^{(\text{SL})}(r) = v_{\text{L}}(r) + v_{\text{H}}^{(\text{v})}(r) + v_{\text{xc}}^{(\text{v}+\text{pcc})}(r).$$

$$|\chi_i\rangle = \left( \varepsilon_i + \frac{1}{2} \nabla^2 - v^{(\text{SL})}(r) \right) |\phi_i^{(\text{PS})}\rangle,$$

$$B_{ij} = \langle \phi_i^{(\text{PS})} | \chi_j \rangle$$

$$|\beta_i\rangle = \sum_j (B^{-1})_{ji} |\chi_j\rangle.$$

It turns out that the following Schrodinger eq. is hold.

$$\left( -\frac{1}{2} \nabla^2 + v^{(\text{SL})}(r) + \hat{v}^{(\text{NL})} \right) |\phi_i^{(\text{PS})}\rangle = \varepsilon_i |\phi_i^{(\text{PS})}\rangle.$$



# Generalized norm-conserving conditions $Q_{ij}$

In the Vanderbilt pseudopotential, B is given by

$$B_{ij} = \int_0^{r_c} dr P_i^{(\text{PS})}(r) \left( \varepsilon_j + \frac{1}{2} \frac{d^2}{dr^2} - \frac{l(l+1)}{2r^2} - v^{(\text{SL})}(r) \right) P_j^{(\text{PS})}(r),$$

$$B_{ji}^* = \int_0^{r_c} dr P_i^{(\text{PS})}(r) \left( \varepsilon_i + \frac{1}{2} \frac{d^2}{dr^2} - \frac{l(l+1)}{2r^2} - v^{(\text{SL})}(r) \right) P_j^{(\text{PS})}(r),$$

Thus, we have

$$B_{ij} - B_{ji}^* = (\varepsilon_j - \varepsilon_i) \int_0^{r_c} dr P_i^{(\text{PS})}(r) P_j^{(\text{PS})}(r) + \frac{1}{2} \int_0^{r_c} dr P_i^{(\text{PS})}(r) P_j''^{(\text{PS})}(r) - \frac{1}{2} \int_0^{r_c} dr P_i''^{(\text{PS})}(r) P_j^{(\text{PS})}(r).$$

By integrating by parts, this can be transformed as

$$\begin{aligned} B_{ij} - B_{ji}^* &= (\varepsilon_j - \varepsilon_i) \langle \phi_i^{(\text{PS})} | \phi_j^{(\text{PS})} \rangle_{r_c} + \frac{1}{2} \left[ P_i^{(\text{PS})}(r) P_j'^{(\text{PS})}(r) \right]_0^{r_c} - \frac{1}{2} \left[ P_i'^{(\text{PS})}(r) P_j^{(\text{PS})}(r) \right]_0^{r_c}, \\ &= (\varepsilon_j - \varepsilon_i) \langle \phi_i^{(\text{PS})} | \phi_j^{(\text{PS})} \rangle_{r_c} + \frac{1}{2} P_i^{(\text{PS})}(r_c) P_j'^{(\text{PS})}(r_c) - \frac{1}{2} P_i'^{(\text{PS})}(r_c) P_j^{(\text{PS})}(r_c). \quad \dots (1) \end{aligned}$$

As well, the similar calculations can be performed for all electron wave functions.

$$0 = (\varepsilon_j - \varepsilon_i) \langle \phi_i^{(\text{AE})} | \phi_j^{(\text{AE})} \rangle_{r_c} + \frac{1}{2} P_i^{(\text{AE})}(r_c) P_j'^{(\text{AE})}(r_c) - \frac{1}{2} P_i'^{(\text{AE})}(r_c) P_j^{(\text{AE})}(r_c). \quad \dots (2)$$

By subtracting (2) from (1), we obtain a relation between B and Q.

$$B_{ij} - B_{ji}^* = (\varepsilon_i - \varepsilon_j) \left( \langle \phi_i^{(\text{AE})} | \phi_j^{(\text{AE})} \rangle_{r_c} - \langle \phi_i^{(\text{PS})} | \phi_j^{(\text{PS})} \rangle_{r_c} \right).$$

# Norm-conserving pseudopotential by MBK

I. Morrion, D.M. Bylander, and L. Kleinman, PRB 47, 6728 (1993).

If  $Q_{ij} = 0$ , the non-local terms can be transformed to a diagonal form.

$$\begin{aligned} V_{\text{NL}} &= \sum_{i,j} B_{ij} |\beta_i\rangle \langle \beta_j|, \\ &= \sum_i \lambda_i |\alpha_i\rangle \langle \alpha_i| \end{aligned}$$

The form is equivalent to that obtained from the Blochl expansion for TM norm-conserving pseudopotentials. Thus, common routines can be utilized for the MBK and TM pseudopotentials, resulting in easiness of the code development.

To satisfy  $Q_{ij}=0$ , pseudofunctions are now given by

$$\phi_i = \phi_{\text{TM},i} + f_i \quad f_i = \sum_{l=0} c_l \left[ r j_l \left( \frac{r}{r_c} u_{li} \right) \right]$$

The coefficients  $\{c\}$  are determined by agreement of derivatives and  $Q_{ij}=0$ . Once a set of  $\{c\}$  is determined,  $\chi$  is given by

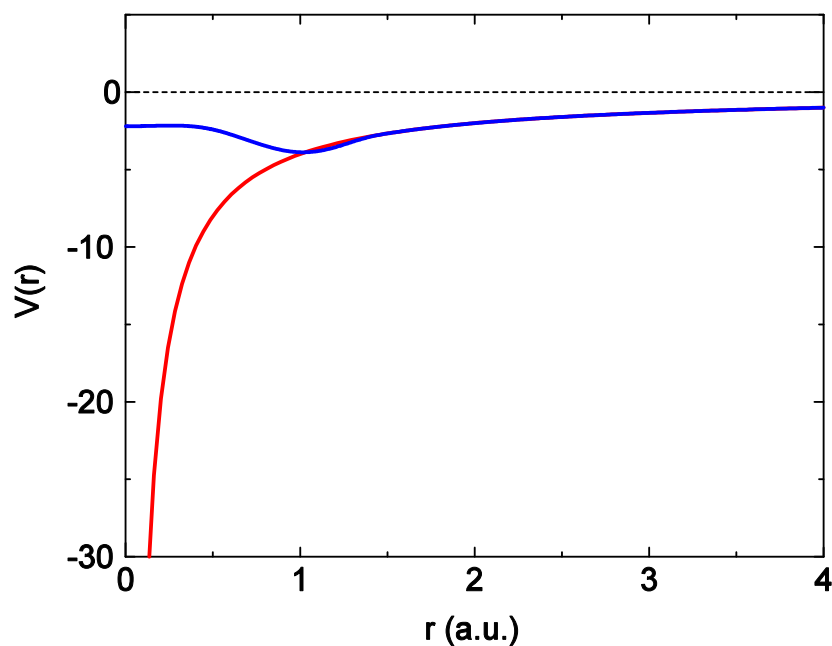
$$\chi_i = V_{\text{TM}}^{(i)} \phi_{\text{TM},i} + \varepsilon_i f_i - V_{\text{loc}} \phi_i - \frac{1}{2} \sum_i c_i \left( \frac{u_{li}}{r_c} \right)^2 \left[ r j_l \left( \frac{r}{r_c} u_{li} \right) \right]$$

# Pseudo-wave function and pseudopotential of carbon atom

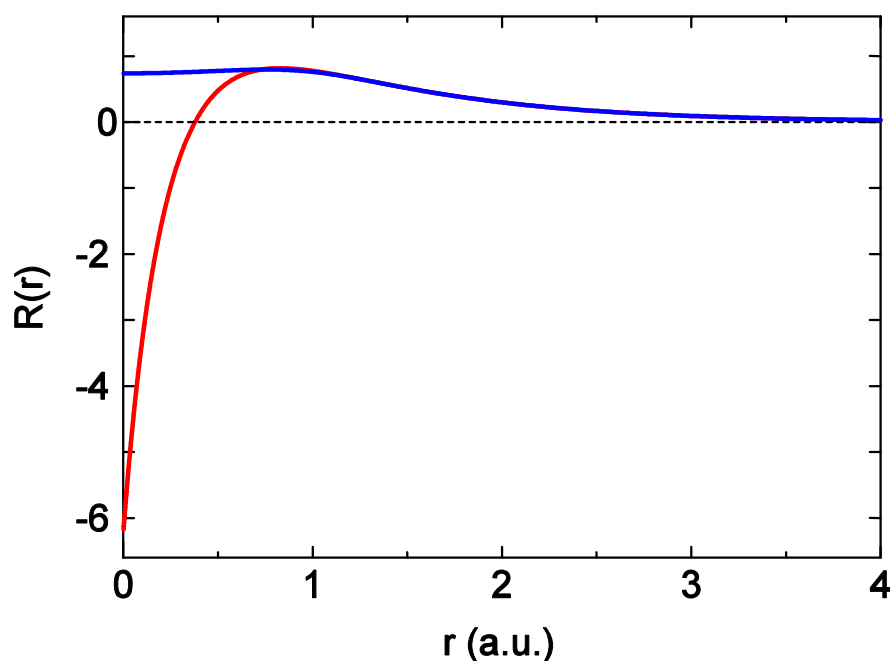
Red: All electron calculation

Blue: Pseudized calculation

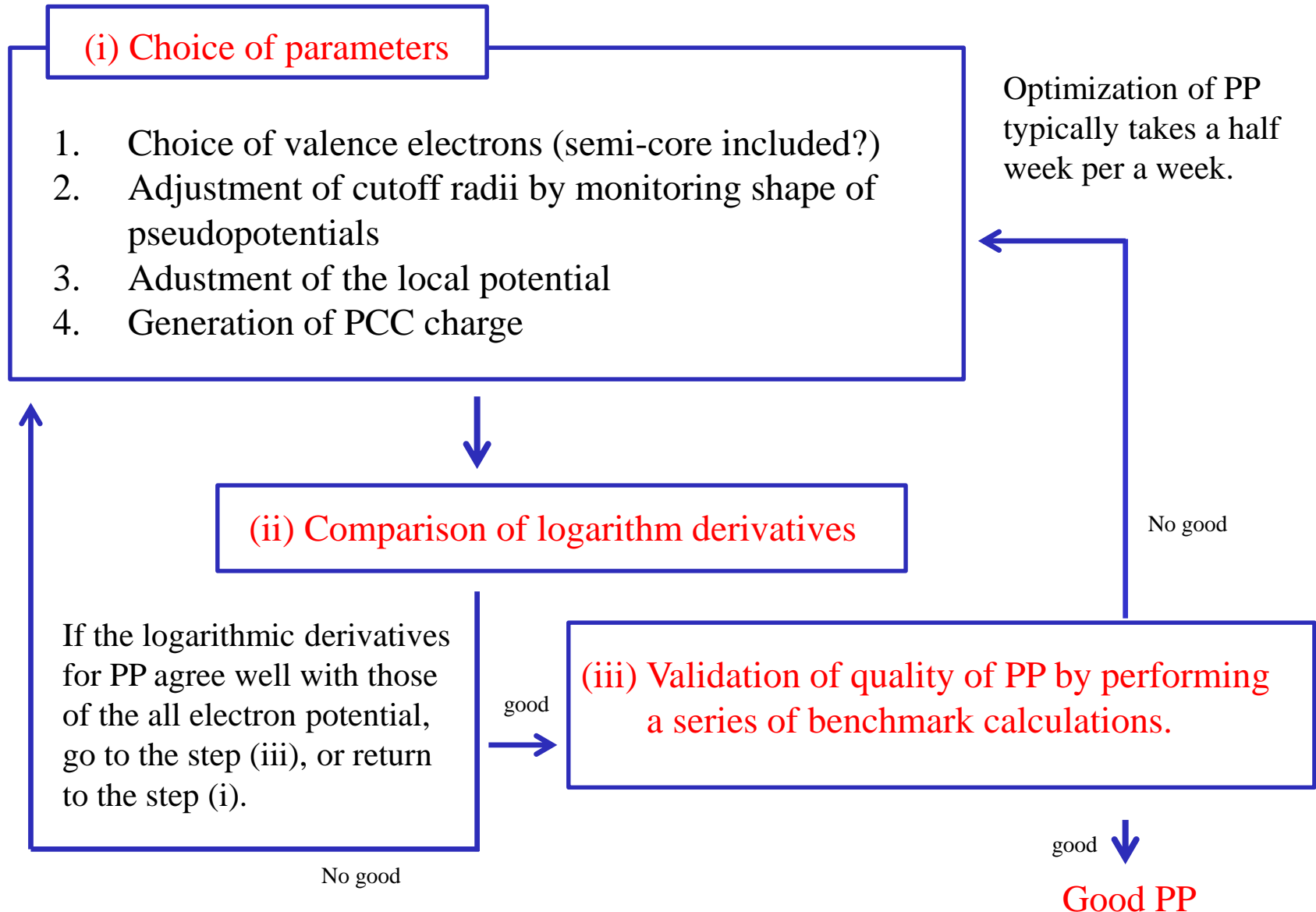
Pseudopotential for C 2s and  $-4/r$



Radial wave function of C 2s

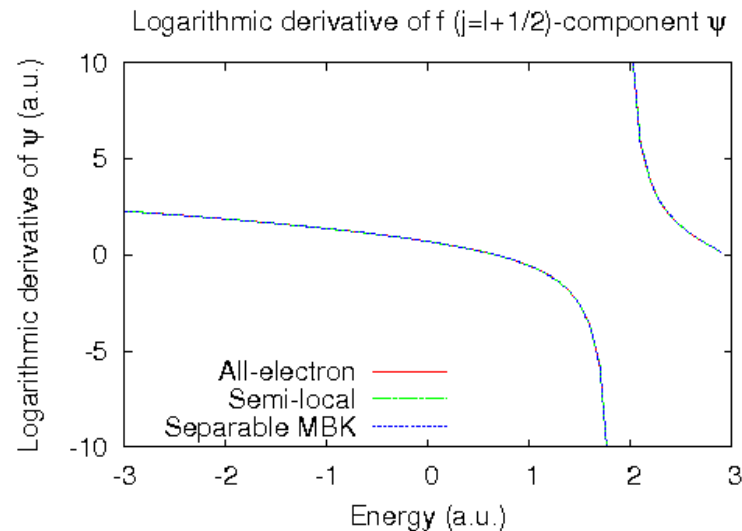
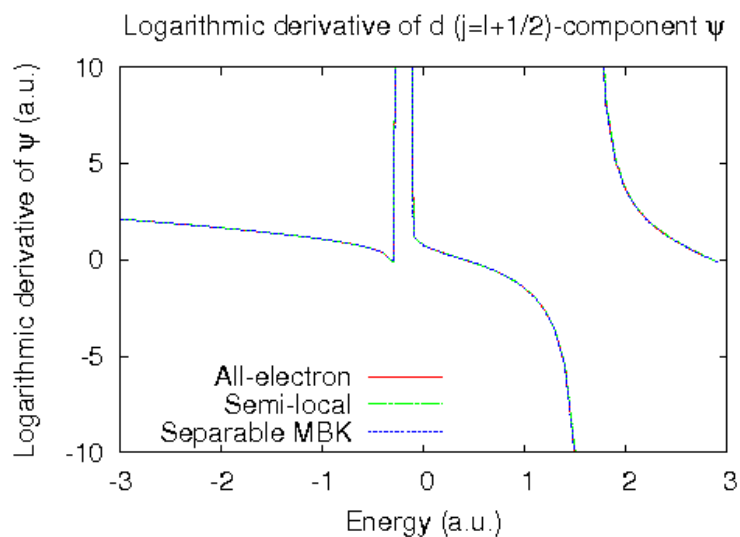
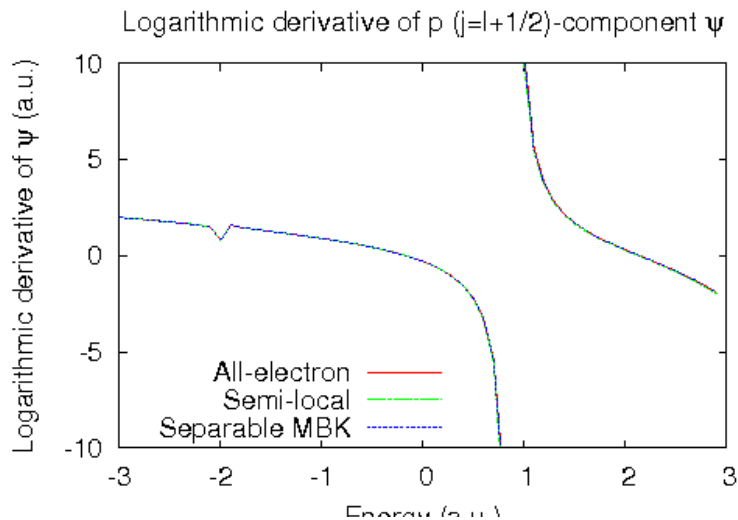
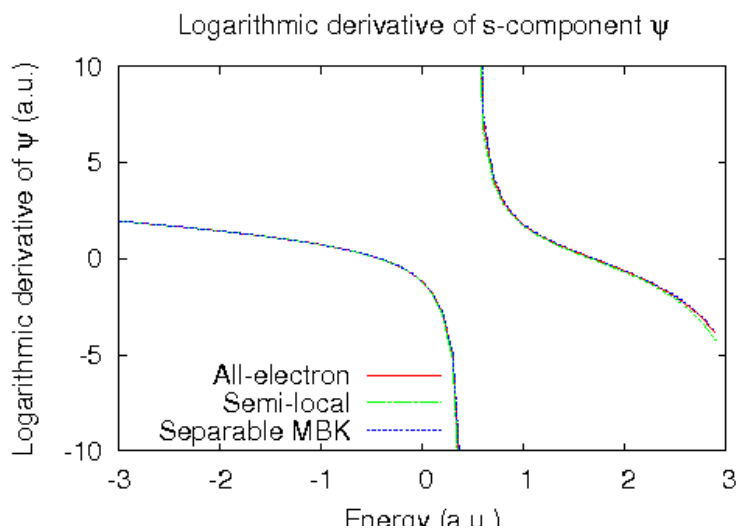


# Optimization of pseudopotentials

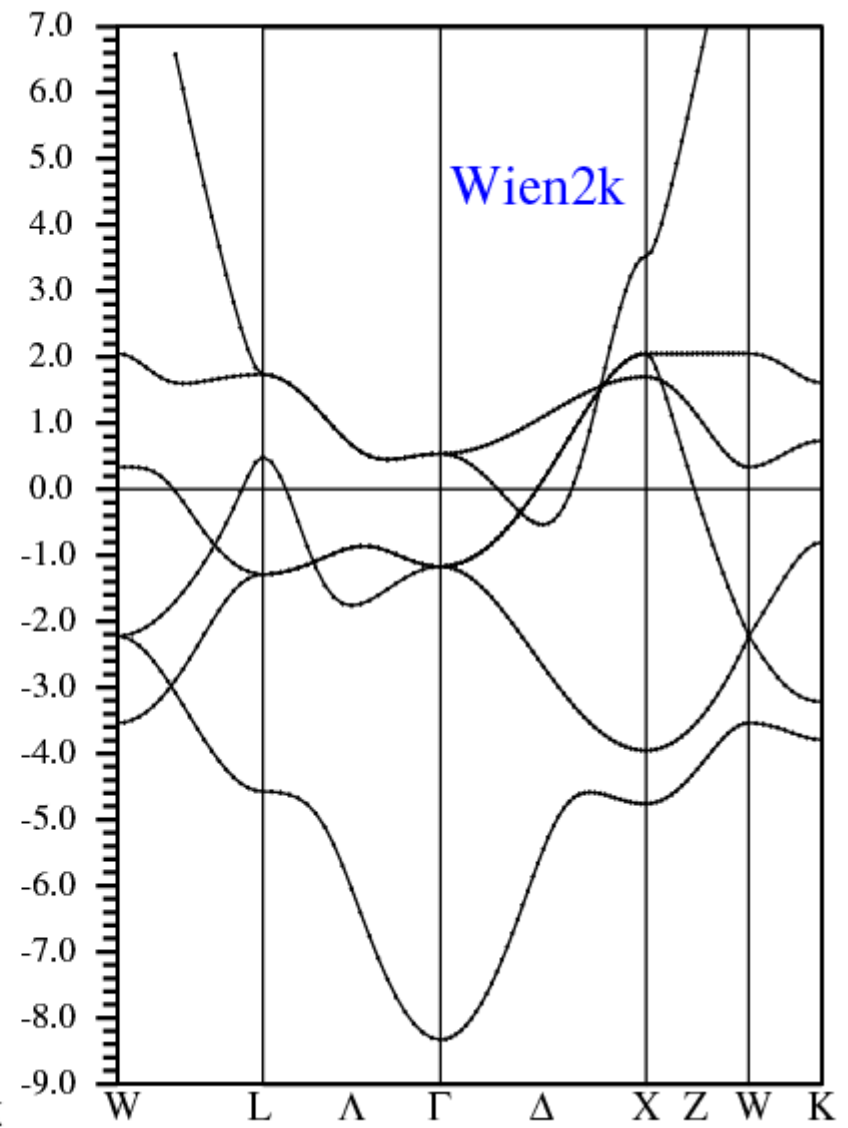
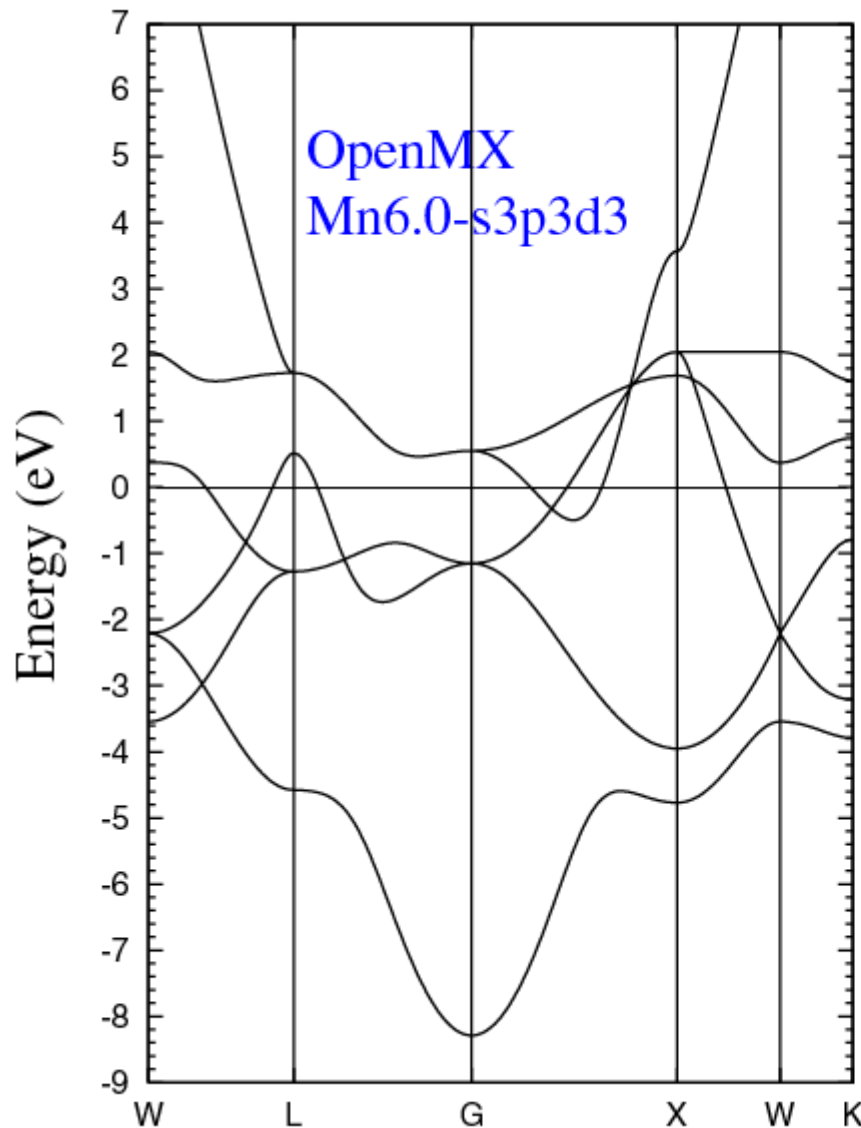


# Comparison of logarithmic derivatives

Logarithmic derivatives of s, p, d, f channels for Mn. The deviation between PP and all electron directly affects the band structure.



# Comparison of band structure for fcc Mn



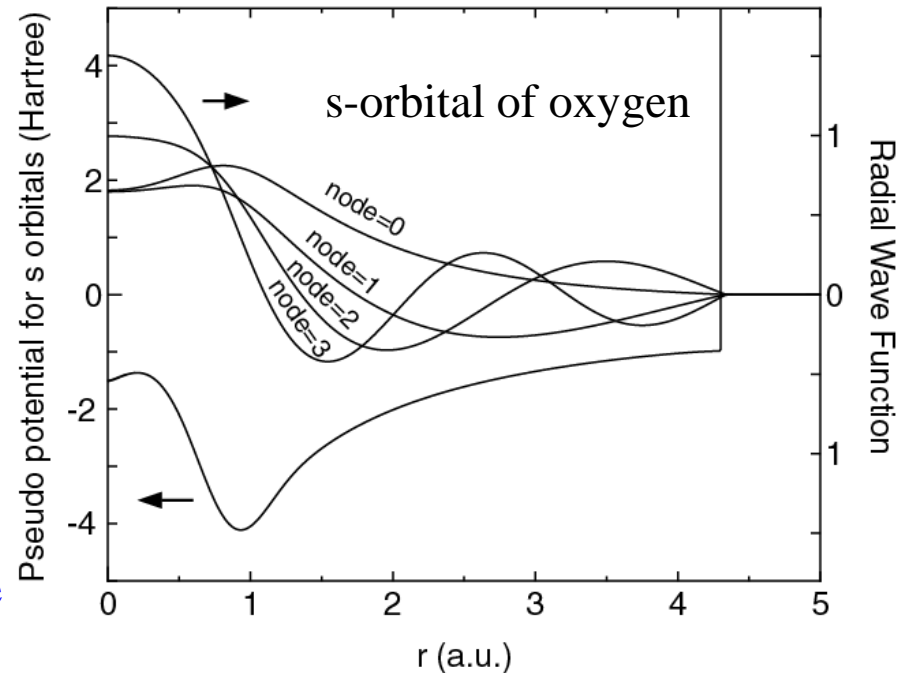
- Total energy
- Pseudopotentials
- **Basis functions**

# Primitive basis functions

1. Solve an atomic Kohn-Sham eq. under a confinement potential:

$$V_{\text{core}}(r) = \begin{cases} -\frac{Z}{r} & \text{for } r \leq r_1 \\ \sum_{n=0}^3 b_n r^n & \text{for } r_1 < r \leq r_c \\ h & \text{for } r_c < r, \end{cases}$$

2. Construct the norm-conserving pseudopotentials.
3. Solve ground and excited states for the the pseudopotential for each L-channel.



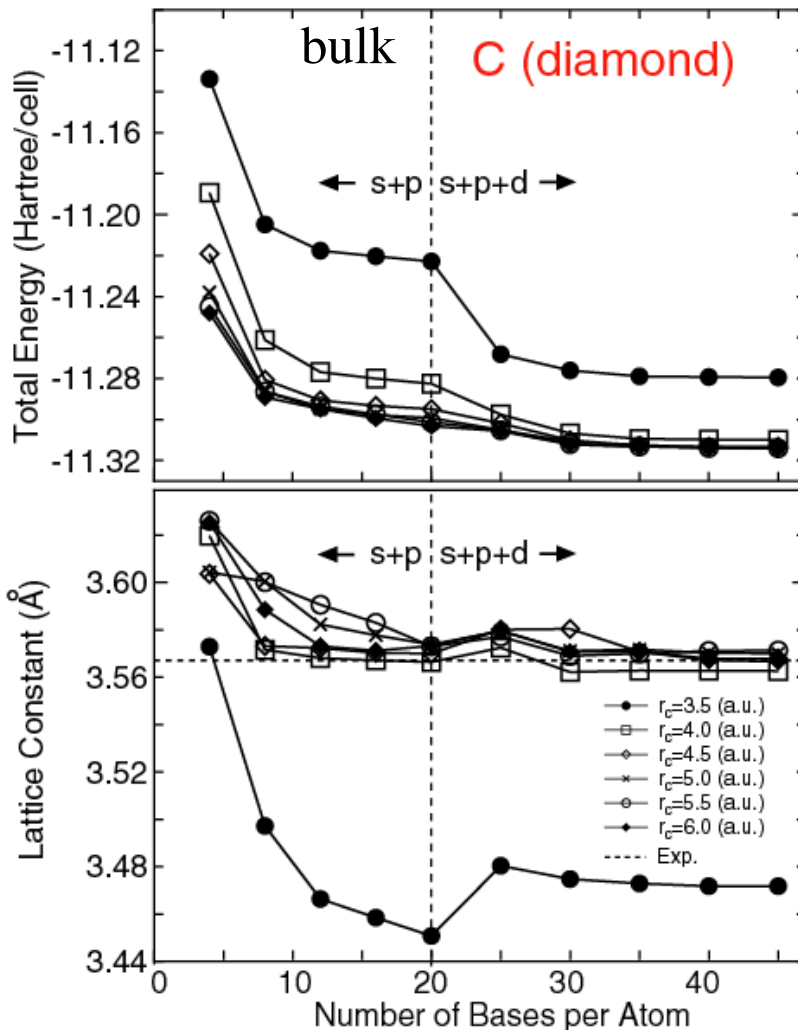
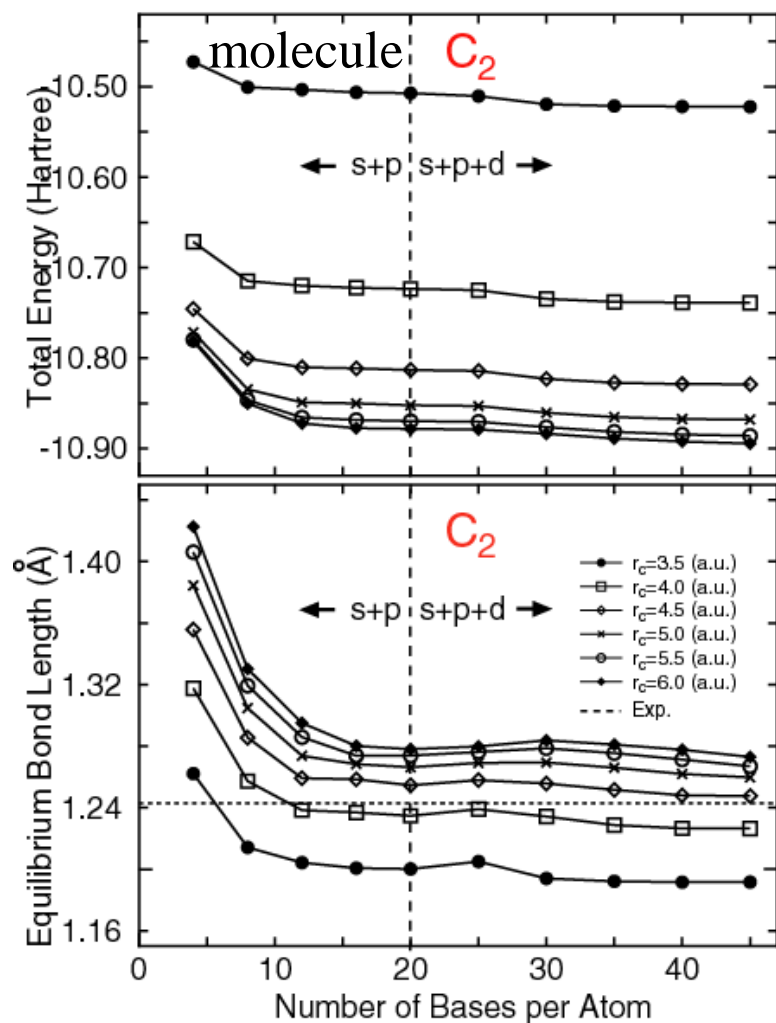
In most cases, the accuracy and efficiency can be controlled by

**Cutoff radius**  
**Number of orbitals**



# Convergence with respect to basis functions

The two parameters can be regarded as variational parameters.



# Benchmark of primitive basis functions

Ground state calculations of dimer using primitive basis functions

Dimer	Expt.	Calc.	Dimer	Expt.	Calc.
H <sub>2</sub> (H4.5- <i>s</i> 2)	$1\Sigma_g^+ a$	$1\Sigma_g^+ (1s_g^2)$	K <sub>2</sub> (K10.0- <i>s</i> 2 <i>p</i> 2)	$1\Sigma_g^+ f$	$1\Sigma_g^+ (3p\pi_g^4 3p\sigma_u^2 4s_g^2)$
He <sub>2</sub> (He7.0- <i>s</i> 2)	$1\Sigma_g^+ b$	$1\Sigma_g^+ (1s_g^2 1s_u^2)$	CaO (Ca7.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 2)	$1\Sigma^+ k$	$1\Sigma^+ (s\sigma^2 s\sigma^2 p\pi^4)$
Li <sub>2</sub> (Li8.0- <i>s</i> 2)	$1\Sigma_g^+ c$	$1\Sigma_g^+ (2s_g^2)$	ScO (Sc7.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 2)	$2\Sigma^+ l$	$2\Sigma^+ (d\pi^4 s\sigma^2 s\sigma^1)$
BeO (Be6.0- <i>s</i> 2 <i>p</i> 2)	$1\Sigma^+ d$	$1\Sigma^+ (s\sigma^2 s\sigma^2 p\pi^4)$	Ti <sub>2</sub> (Ti7.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 2)	$3\Delta_g m$	$3\Delta_g (4s\sigma_g^2 3d\sigma_g^1 3d\pi_u^4 3d\delta_g^1)$
B <sub>2</sub> (B5.5- <i>s</i> 2 <i>p</i> 2)	$3\Sigma_g^- e$	$3\Sigma_g^- (2s\sigma_g^2 2s\sigma_u^2 2\pi_u^2)$	V <sub>2</sub> (V7.5- <i>s</i> 2 <i>p</i> 2 <i>d</i> 2)	$3\Sigma_g^- n$	$1\Sigma_g^+ (4s\sigma_g^2 3d\sigma_g^2 3d\pi_u^4 3d\delta_g^2)$
C <sub>2</sub> (C5.0- <i>s</i> 2 <i>p</i> 2)	$1\Sigma_g^+ f$	$1\Sigma_g^+ (2s\sigma_g^2 2s\sigma_u^2 2p\pi_u^4)$	V <sub>2</sub> (V7.5- <i>s</i> 4 <i>p</i> 4 <i>d</i> 4 <i>f</i> 2)	$3\Sigma_g^- n$	$3\Sigma_g^- (4s\sigma_g^2 3d\sigma_g^2 3d\pi_u^4 3d\delta_g^2)$
N <sub>2</sub> (N5.0- <i>s</i> 2 <i>p</i> 2)	$1\Sigma_g^+ f$	$1\Sigma_g^+ (2s\sigma_u^2 2p\pi_u^4 2p\sigma_g^2)$	Cr <sub>2</sub> (Cr7.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 2)	$1\Sigma^+ o$	$1\Sigma^+ (4s\sigma_g^2 3d\sigma_g^2 3d\pi_u^4 3d\delta_g^4)$
O <sub>2</sub> (O5.0- <i>s</i> 2 <i>p</i> 2)	$3\Sigma_g^- f$	$3\Sigma_g^- (2p\sigma_g^2 2p\pi_u^4 2p\pi_g^2)$	MnO (Mn7.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 2)	$6\Sigma^+ p$	$6\Sigma^+ (d\sigma^1 d\pi^4 d\delta^2 d\pi^{*2})$
F <sub>2</sub> (F5.0- <i>s</i> 2 <i>p</i> 2)	$1\Sigma_g^+ f$	$1\Sigma_g^+ (2p\sigma_g^2 2p\pi_u^4 2p\pi_g^4)$	Fe <sub>2</sub> (Fe7.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 2)	$7\Delta_u q$	$7\Delta_u (4s\sigma_g^2 3d\sigma_g^2 3d\sigma_u^1 3d\pi_u^4 3d\pi_g^2 3d\delta_g^3 3d\delta_u^2)$
Ne <sub>2</sub> (Ne7.0- <i>s</i> 2 <i>p</i> 2)	$1\Sigma_g^+ g$	$1\Sigma_g^+ (2p\pi_u^4 2p\pi_g^4 2p\sigma_u^2)$	Co <sub>2</sub> (Co7.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 2)		$5\Delta_g (4s\sigma_g^2 3d\sigma_g^2 3d\sigma_u^1 3d\pi_u^4 3d\pi_g^2 3d\delta_g^4 3d\delta_u^3)$
Na <sub>2</sub> (Na9.0- <i>s</i> 2 <i>p</i> 2)	$1\Sigma_g^+ f$	$1\Sigma_g^+ (2p\pi_g^4 2p\sigma_u^2 3s\sigma_g^2)$	Ni <sub>2</sub> (Ni7.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 2)	$\Omega r$	$3\Sigma_g^- (4s\sigma_g^2 3d\sigma_g^2 3d\sigma_u^2 3d\pi_u^4 3d\pi_g^2 3d\delta_g^4 3d\delta_u^4)$
MgO (Mg7.0- <i>s</i> 2 <i>p</i> 2)	$1\Sigma^+ h$	$1\Sigma^+ (s\sigma^2 s\sigma^2 p\pi^4)$	Cu <sub>2</sub> (Cu7.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 2)	$1\Sigma_g^+ s$	$1\Sigma_g^+ (4s\sigma_g^2 3d\sigma_g^2 3d\sigma_u^2 3d\pi_u^4 3d\pi_g^4 3d\delta_g^4 3d\delta_u^4)$
Al <sub>2</sub> (Al6.5- <i>s</i> 2 <i>p</i> 2)	$3\Pi_u i$	$3\Sigma_g^- (3s\sigma_g^2 3s\sigma_u^2 3p\pi_u^2)$	ZnH (Zn7.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 2)	$2\Sigma_g^+ t$	$2\Sigma_g^+ (s\sigma^2 s\sigma^{*1} d\sigma^2 d\pi^4 d\delta^4)$
Al <sub>2</sub> (Al6.5- <i>s</i> 4 <i>p</i> 4 <i>d</i> 2)	$3\Pi_u i$	$3\Sigma_g^- (3s\sigma_g^2 3s\sigma_u^2 3p\pi_u^2)$	GaH (Ga7.0- <i>s</i> 2 <i>p</i> 2)	$1\Sigma^+ u$	$1\Sigma^+ (s\sigma^2 s\sigma^{*2})$
Si <sub>2</sub> (Si6.5- <i>s</i> 2 <i>p</i> 2)	$3\Sigma_g^- f$	$3\Pi_u (3s\sigma_u^2 3s\sigma_g^1 3p\pi_u^3)$	GeO (Ge7.0- <i>s</i> 2 <i>p</i> 2)	$1\Sigma^+ f$	$1\Sigma^+ (ss\sigma^2 sp\sigma^2 pp\pi^4 pp\sigma^2)$
Si <sub>2</sub> (Si6.5- <i>s</i> 2 <i>p</i> 2 <i>d</i> 1)	$3\Sigma_g^- f$	$3\Sigma_g^- (3s\sigma_u^2 3p\pi_u^2 3s\sigma_g^2)$	As <sub>2</sub> (As7.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 1)	$1\Sigma_g^+ f$	$1\Sigma_g^+ (4s\sigma_g^2 4s\sigma_u^2 4p\sigma_g^2 4p\pi_u^4)$
P <sub>2</sub> (P6.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 1)	$1\Sigma_g^+ f$	$1\Sigma_g^+ (3s\sigma_u^2 3p\sigma_g^2 3p\pi_u^4)$	Se <sub>2</sub> (Se7.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 1)	$3\Sigma_g^- f$	$3\Sigma_g^- (4s\sigma_g^2 4s\sigma_u^2 4p\sigma_g^2 4p\pi_u^4 4p\pi_g^2)$
S <sub>2</sub> (S6.0- <i>s</i> 2 <i>p</i> 2)	$3\Sigma_g^- f$	$3\Sigma_g^- (3p\sigma_g^2 3p\pi_u^4 3p\pi_g^2)$	Br <sub>2</sub> (Br7.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 1)	$1\Sigma_g^+ f$	$1\Sigma_g^+ (4s\sigma_g^2 4s\sigma_u^2 4p\sigma_g^2 4p\pi_u^4 4p\pi_g^4)$
Cl <sub>2</sub> (Cl6.0- <i>s</i> 2 <i>p</i> 2 <i>d</i> 2)	$1\Sigma_g^+ f$	$1\Sigma_g^+ (3p\sigma_g^2 3p\pi_u^4 3p\pi_g^4)$	Kr <sub>2</sub> (Kr7.0- <i>s</i> 2 <i>p</i> 2)	$1\Sigma_g^+ v$	$1\Sigma_g^+ (4s\sigma_g^2 4s\sigma_u^2 4p\sigma_g^2 4p\sigma_u^2 4p\pi_u^4 4p\pi_g^4)$
Ar <sub>2</sub> (Ar7.0- <i>s</i> 2 <i>p</i> 2)	$1\Sigma_g^+ j$	$1\Sigma_g^+ (3p\pi_u^4 3p\pi_g^4 3p\sigma_u^2)$			

All the successes and failures by the LDA are reproduced by the modest size of basis functions (DNP in most cases)

# Variational optimization of basis functions

One-particle wave functions

$$\psi_{\mu}(\mathbf{r}) = \sum_{i\alpha} c_{\mu,i\alpha} \phi_{i\alpha}(\mathbf{r} - \mathbf{r}_i)$$

Contracted orbitals

$$\phi_{i\alpha}(\mathbf{r}) = \sum_q a_{i\alpha q} \chi_{i\eta}(\mathbf{r})$$

The variation of  $E$  with respect to  $c$  with fixed  $a$  gives

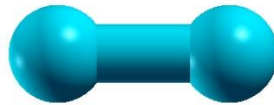
$$\partial E_{\text{tot}} / \partial c_{\mu,i\alpha} = 0 \quad \rightarrow \quad \sum_{j\beta} \langle \phi_{i\alpha} | \hat{H} | \phi_{j\beta} \rangle c_{\mu,j\beta} = \varepsilon_{\mu} \sum_{j\beta} \langle \phi_{i\alpha} | \phi_{j\beta} \rangle c_{\mu,j\beta}$$

Regarding  $c$  as dependent variables on  $a$  and assuming KS eq. is solved self-consistently with respect to  $c$ , we have

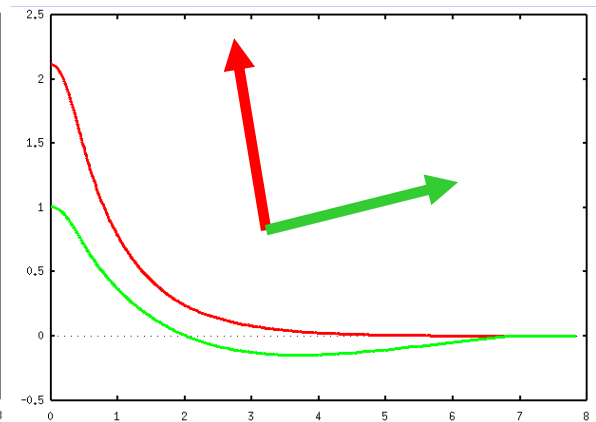
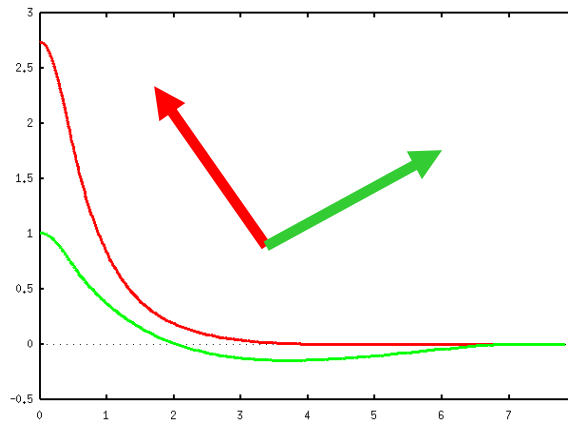
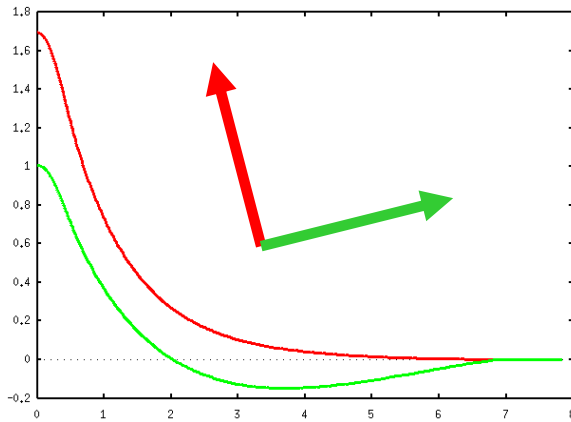
$$\begin{aligned} \frac{\partial E_{\text{tot}}}{\partial a_{i\alpha q}} &= \frac{\delta E_{\text{tot}}}{\delta \rho(\mathbf{r})} \frac{\delta \rho(\mathbf{r})}{\delta a_{i\alpha q}} \\ &= 2 \sum_{j\beta} (\Theta_{i\alpha,j\beta} \langle \chi_{i\eta} | \hat{H} | \phi_{j\beta} \rangle - E_{i\alpha,j\beta} \langle \chi_{i\eta} | \phi_{j\beta} \rangle) \end{aligned}$$

# Optimization of basis functions

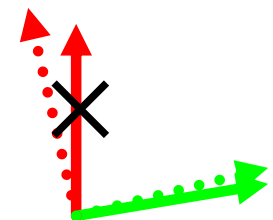
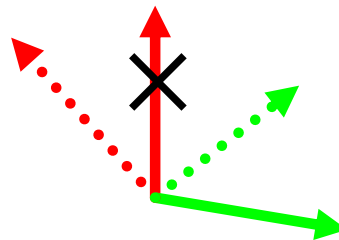
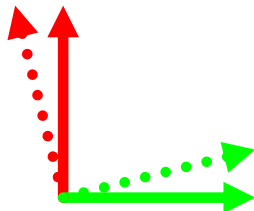
1. Choose typical chemical environments



2. Optimize variationally the radial functions



3. Rotate a set of optimized orbitals within the subspace, and discard the redundant functions



# Database of optimized VPS and PAO

## Database (2013) of optimized VPS and PAO

The database (2013) of fully relativistic pseudopotentials (VPS) and pseudo-atomic orbitals (PAO), generated by ADPACK, which could be an input data of program package, OpenMX. The data of elements with the underline are currently available. When you use these data, VPS and PAO, in the program package, OpenMX, then copy them to the directory, openmx\*/DFT\_DATA13/VPS/ and openmx\*/DFT\_DATA13/PAO/, respectively. The delta factor of OpenMX with the database (2013) is found at [here](#).

Public release of optimized and well tested VPS and PAO so																			
that users can easily start their calculations.																			
<u>E</u>																			<u>He</u>
<u>H</u>																			
<u>Li</u>	<u>Be</u>											<u>B</u>	<u>C</u>	<u>N</u>	<u>O</u>	<u>F</u>		<u>Ne</u>	
<u>Na</u>	<u>Mg</u>											<u>Al</u>	<u>Si</u>	<u>P</u>	<u>S</u>	<u>Cl</u>		<u>Ar</u>	
<u>K</u>	<u>Ca</u>	<u>Sc</u>	<u>Ti</u>	<u>V</u>	<u>Cr</u>	<u>Mn</u>	<u>Fe</u>	<u>Co</u>	<u>Ni</u>	<u>Cu</u>	<u>Zn</u>	<u>Ga</u>	<u>Ge</u>	<u>As</u>	<u>Se</u>	<u>Br</u>		<u>Kr</u>	
<u>Rb</u>	<u>Sr</u>	<u>Y</u>	<u>Zr</u>	<u>Nb</u>	<u>Mo</u>	<u>Tc</u>	<u>Ru</u>	<u>Rh</u>	<u>Pd</u>	<u>Ag</u>	<u>Cd</u>	<u>In</u>	<u>Sn</u>	<u>Sb</u>	<u>Te</u>	<u>I</u>		<u>Xe</u>	
<u>Cs</u>	<u>Ba</u>	<u>L</u>	<u>Hf</u>	<u>Ta</u>	<u>W</u>	<u>Re</u>	<u>Os</u>	<u>Ir</u>	<u>Pt</u>	<u>Au</u>	<u>Hg</u>	<u>Tl</u>	<u>Pb</u>	<u>Bi</u>	<u>Po</u>	<u>At</u>		<u>Rn</u>	
<u>Fr</u>	<u>Ra</u>	<u>A</u>																	
	<u>L</u>	<u>La</u>	<u>Ce</u>	<u>Pr</u>	<u>Nd</u>	<u>Pm</u>	<u>Sm</u>	<u>Eu</u>	<u>Gd</u>	<u>Tb</u>	<u>Dy</u>	<u>Ho</u>	<u>Er</u>	<u>Tm</u>	<u>Yb</u>		<u>Lu</u>		
	<u>A</u>	<u>Ac</u>	<u>Th</u>	<u>Pa</u>	<u>U</u>	<u>Np</u>	<u>Pu</u>	<u>Am</u>	<u>Cm</u>	<u>Bk</u>	<u>Cf</u>	<u>Es</u>	<u>Fm</u>	<u>Md</u>	<u>No</u>		<u>Lr</u>		

# Reproducibility in DFT calcs

## RESEARCH ARTICLE

Science 351, aad3000 (2016)

DFT METHODS

## Reproducibility in density functional theory calculations of solids

Kurt Lejaeghere,<sup>1\*</sup> Gustav Bihlmayer,<sup>2</sup> Torbjörn Björkman,<sup>3,4</sup> Peter Blaha,<sup>5</sup> Stefan Blügel,<sup>2</sup> Volker Blum,<sup>6</sup> Damien Caliste,<sup>7,8</sup> Ivano E. Castelli,<sup>9</sup> Stewart J. Clark,<sup>10</sup> Andrea Dal Corso,<sup>11</sup> Stefano de Gironcoli,<sup>11</sup> Thierry Deutsch,<sup>7,8</sup> John Kay Dewhurst,<sup>12</sup> Igor Di Marco,<sup>13</sup> Claudia Draxl,<sup>14,15</sup> Marcin Dułak,<sup>16</sup> Olle Eriksson,<sup>13</sup> José A. Flores-Livas,<sup>12</sup> Kevin F. Garrity,<sup>17</sup> Luigi Genovese,<sup>7,8</sup> Paolo Giannozzi,<sup>18</sup> Matteo Giantomassi,<sup>19</sup> Stefan Goedecker,<sup>20</sup> Xavier Gonze,<sup>19</sup> Oscar Grånäs,<sup>13,21</sup> E. K. U. Gross,<sup>12</sup> Andris Gulans,<sup>14,15</sup> François Gygi,<sup>22</sup> D. R. Hamann,<sup>23,24</sup> Phil J. Hasnip,<sup>25</sup> N. A. W. Holzwarth,<sup>26</sup> Diana Iuşan,<sup>13</sup> Dominik B. Jochym,<sup>27</sup> François Jollet,<sup>28</sup> Daniel Jones,<sup>29</sup> Georg Kresse,<sup>30</sup> Klaus Koepernik,<sup>31,32</sup> Emine Küçükbenli,<sup>9,11</sup> Yaroslav O. Kvashnin,<sup>13</sup> Inka L. M. Locht,<sup>13,33</sup> Sven Lubeck,<sup>14</sup> Martijn Marsman,<sup>30</sup> Nicola Marzari,<sup>9</sup> Ulrike Nitzsche,<sup>31</sup> Lars Nordström,<sup>13</sup> Taisuke Ozaki,<sup>34</sup> Lorenzo Paulatto,<sup>35</sup> Chris J. Pickard,<sup>36</sup> Ward Poelmans,<sup>1,37</sup> Matt I. J. Probert,<sup>25</sup> Keith Refson,<sup>38,39</sup> Manuel Richter,<sup>31,32</sup> Gian-Marco Rignanese,<sup>19</sup> Santanu Saha,<sup>20</sup> Matthias Scheffler,<sup>15,40</sup> Martin Schlupf,<sup>22</sup> Karlheinz Schwarz,<sup>5</sup> Sangeeta Sharma,<sup>12</sup> Francesca Tavazza,<sup>17</sup> Patrik Thunström,<sup>41</sup> Alexandre Tkatchenko,<sup>15,42</sup> Marc Torrent,<sup>28</sup> David Vanderbilt,<sup>23</sup> Michiel J. van Setten,<sup>19</sup> Veronique Van Speybroeck,<sup>1</sup> John M. Wills,<sup>43</sup> Jonathan R. Yates,<sup>29</sup> Guo-Xu Zhang,<sup>44</sup> Stefaan Cottenier<sup>1,45\*</sup>

15 codes

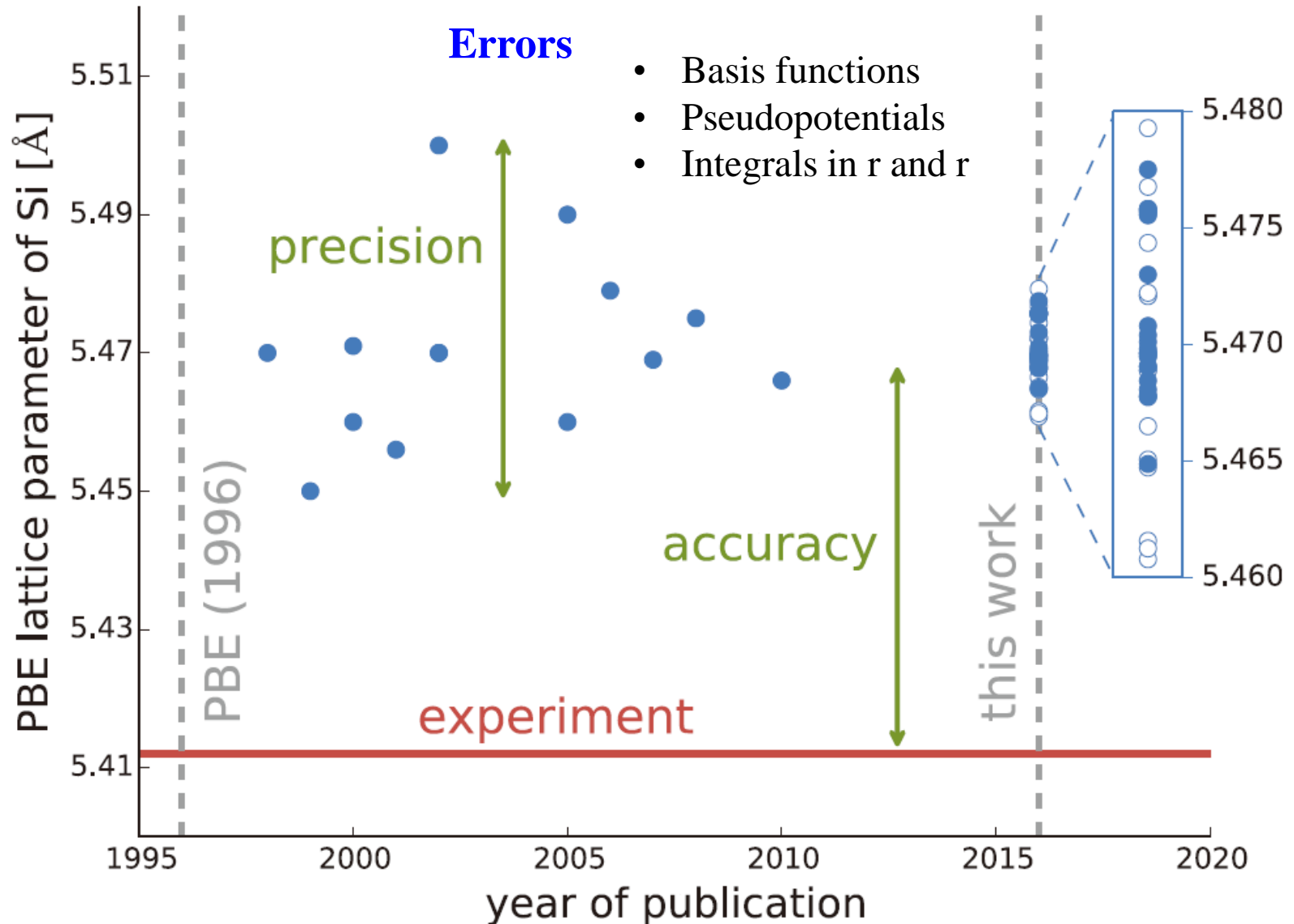
69 researchers

71 elemental bulks

GGA-PBE

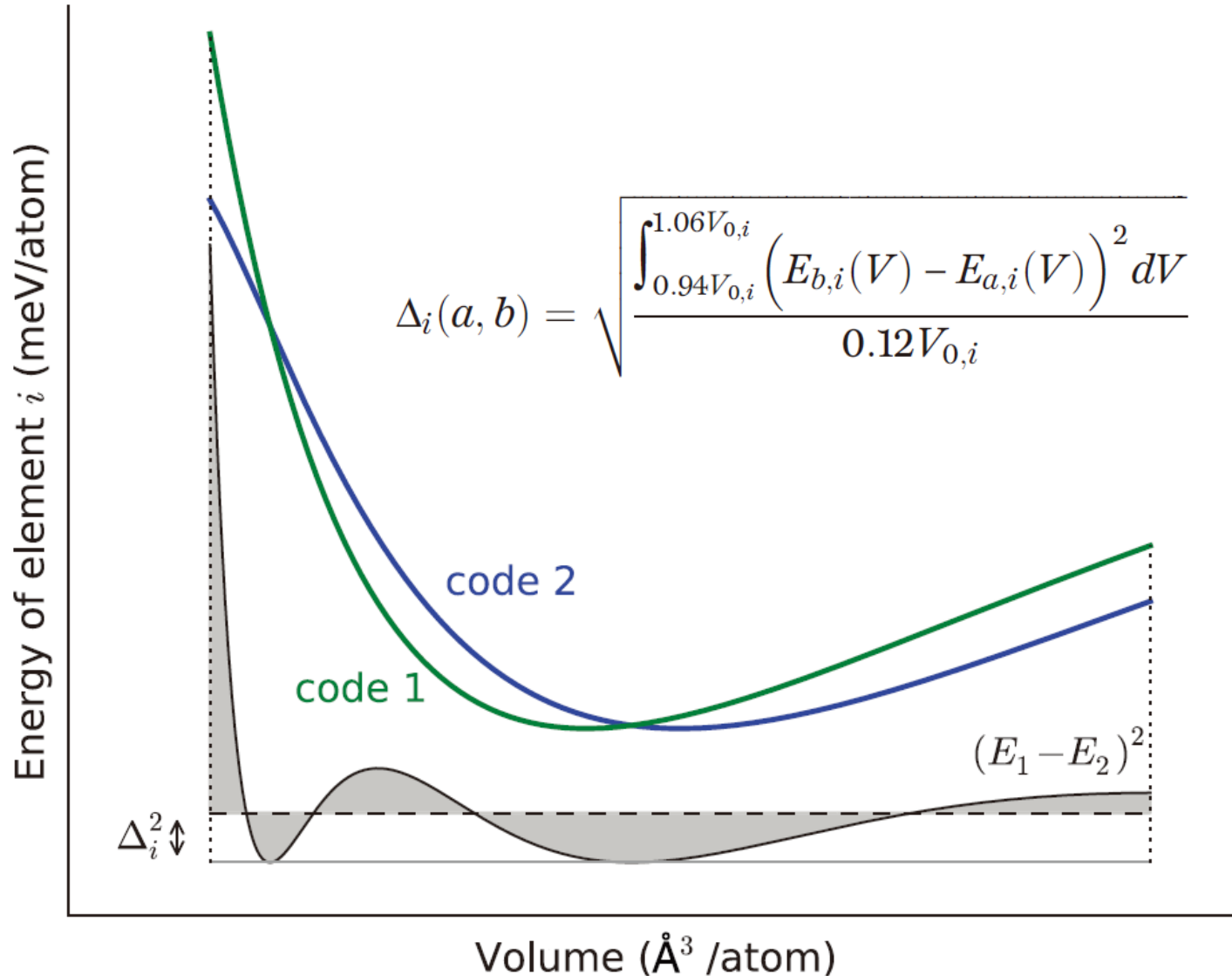
Scalar relativistic

# PBE lattice constant of Si



# $\Delta$ -gauge

A way of comparing accuracy of codes





# Evaluation of GGA-PBE By $\Delta$ -gauge

In comparison of GGA-PBE with Expts. of 58 elements, the mean  $\Delta$ -gauge is 23.5meV/atom.

H																	He
Li	Be	$\Delta(\text{exp}) = 23.5 \text{ meV/atom}$										B	C	N	O	F	Ne
0.4													17.6				17.1
Na	Mg											Al	Si	P	S	Cl	Ar
0.2	1.5											3.9	13.7	19.0	44.0	46.1	38.6
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
1.4	3.0	1.7	3.0	13.1	1.6	47.7	10.6	3.3	6.2	13.5	7.6		28.2	17.1	11.1	22.3	54.6
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
1.3	2.4	0.7	8.2	14.8	26.5		30.4	44.6	49.1	39.3	34.0	23.6	40.7	25.1	9.9	68.1	59.8
Cs	Ba	Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
4.0	2.5		2.8	16.8	35.6	29.9	65.2	50.7	54.9	64.0		42.4	32.9	32.0			

# Comparison of codes by $\Delta$ -gauge

		AE							
		average $\langle \Delta \rangle$	Elk	exciting	FHI-aims/tier2	FLEUR	FPLO/T+F+s	RSpt	WIEN2k/acc
AE	Elk	0.6		0.3	0.3	0.6	1.0	0.9	0.3
	exciting	0.5	0.3		0.1	0.5	0.9	0.8	0.2
	FHI-aims/tier2	0.5	0.3	0.1		0.5	0.9	0.8	0.2
	FLEUR	0.6	0.6	0.5	0.5		0.8	0.6	0.4
	FPLO/T+F+s	0.9	1.0	0.9	0.9	0.8		0.9	0.9
	RSpt	0.8	0.9	0.8	0.8	0.6	0.9		0.8
PAW	WIEN2k/acc	0.5	0.3	0.2	0.2	0.4	0.9	0.8	
	GBRV12/ABINIT	0.9	0.9	0.8	0.8	0.9	1.3	1.1	0.8
	GPW09/ABINIT	1.4	1.3	1.3	1.3	1.3	1.7	1.5	1.3
	GPW09/GPAW	1.6	1.5	1.5	1.5	1.5	1.8	1.7	1.5
	JTH02/ABINIT	0.6	0.6	0.6	0.6	0.6	0.9	0.7	0.5
	PS1b100/QE	0.9	0.9	0.8	0.8	0.8	1.3	1.1	0.8
USPP	VASP GW2015/VASP	0.6	0.4	0.4	0.4	0.6	1.0	0.8	0.3
	GBRV14/CASTEP	1.1	1.1	1.1	1.0	1.0	1.4	1.3	1.0
	GBRV14/QE	1.1	1.0	1.0	0.9	1.0	1.4	1.3	1.0
	OTFG9/CASTEP	0.7	0.4	0.5	0.5	0.7	1.0	1.0	0.5
	SSSP/QE	0.5	0.4	0.3	0.3	0.5	0.9	0.8	0.3
	Vdb2/DACAPO	6.3	6.3	6.3	6.3	6.3	6.4	6.5	6.2
NCP	FHI98pp/ABINIT	13.3	13.5	13.4	13.4	13.2	13.0	13.2	13.4
	HGH/ABINIT	2.2	2.2	2.2	2.2	2.0	2.3	2.2	2.1
	HGH-NLCC/BigDFT	1.1	1.1	1.1	1.1	1.0	1.2	1.1	1.0
	MBK2013/OpenMX	2.0	2.1	2.1	2.1	1.9	1.8	1.8	2.0
	ONCVSP (PD0.1)/ABINIT	0.7	0.7	0.7	0.7	0.6	1.0	0.8	0.6
	ONCVSP (SG15) 1/QE	1.4	1.4	1.3	1.3	1.3	1.6	1.5	1.3
ONCVSP (SG15) 2/CASTEP		1.4	1.4	1.4	1.4	1.3	1.6	1.5	1.4

The mean  $\Delta$ -gauge of OpenMX is 2.0meV/atom.

# Practical guide to OpenMX calculations

- Choice of cutoff energy
- Calculations of energy curves
- SCF calculations
- How to choose basis functions
- Work functions and floating states
- Overcompleteness
- Restarting
- Outputting in a binary mode

# Choice of cutoff energy

`scf.energycutoff`

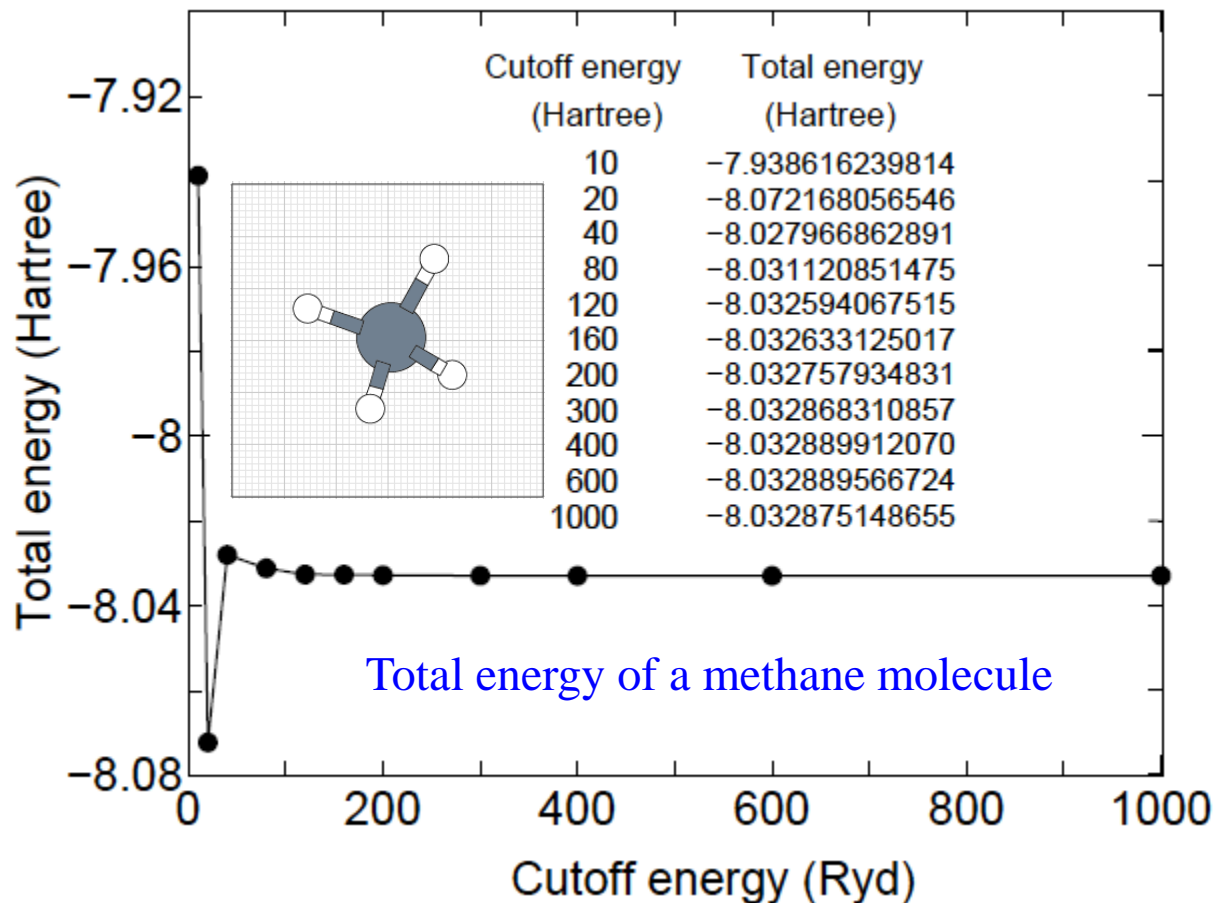
200 # default=150 Ryd

The FFT grid is used to discretize real space and calculate  $E_{\delta ee}$ ,  $E_{xc}$ , and can be specified by `scf.energycutoff`.

In most cases, 200 Ryd is enough to get convergence.

However, large cutoff energy (300-400 Ryd) has be used for cases such as use of pseudopotentials with deep semi-core states.

Memory requiment  
 $O(E^{3/2})$



# Choice of cutoff energy

Geometry optimization of H<sub>2</sub>O

Dependency of optimized structure of H<sub>2</sub>O on scf.energycutoff. It turns out that 180Ryd. is enough to reach the convergence.

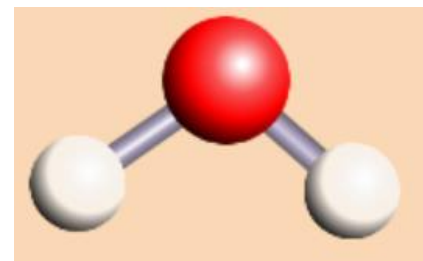


Table 1: Convergence of structural parameters, dipole moment of a water molecule with respect to the cutoff energy. The input file is 'H2O.dat' in the directory 'work'.

Ecut(Ryd)	r(H-O) (Å)	∠ (H-O-H) (deg)	Dipole moment (Debye)
60	0.970	103.4	1.838
90	0.971	103.7	1.829
120	0.971	103.7	1.832
150	0.971	103.6	1.829
180	0.971	103.6	1.833
Exp.	0.957	104.5	1.85

# Volume vs. Energy curves

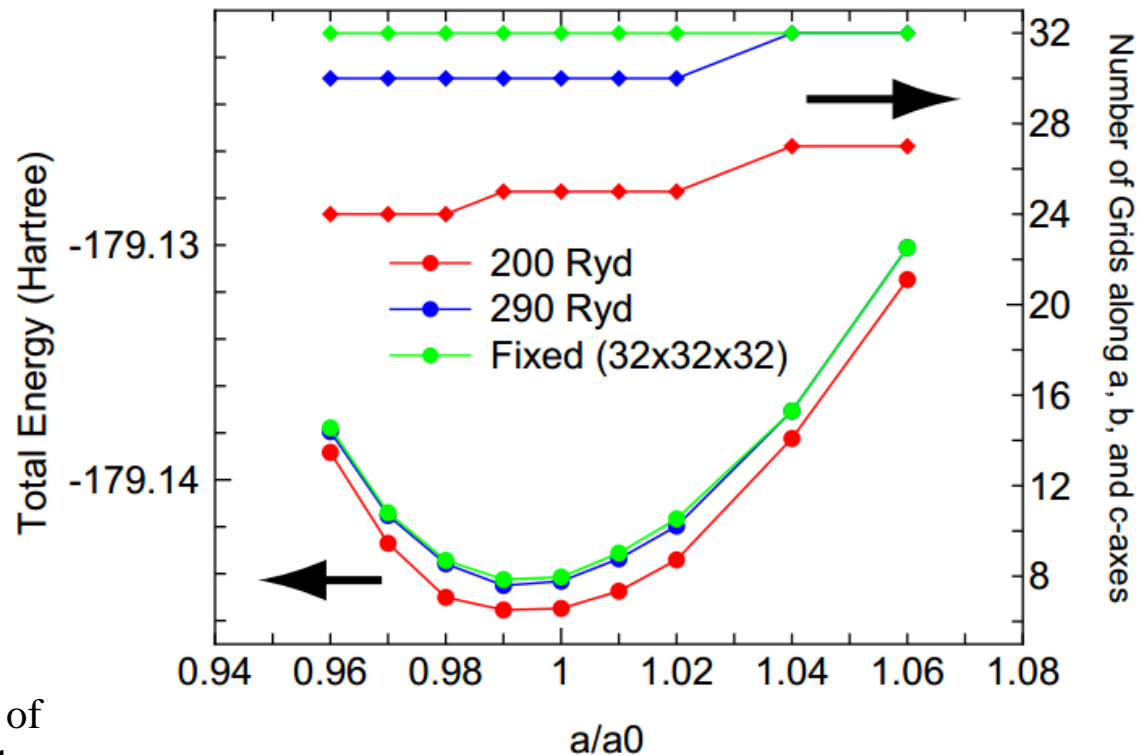
The following keywords are available to calculate energy curves.

MD.Type	EvsLC	#
MD.EvsLC.Step	0.4	# default=0.4%
MD.maxIter	32	# default=1

When the energy curve for bulk system is calculated as a function of the lattice parameter, a sudden change of the number of real space grids is a serious problem which produces an erratic discontinuity on the energy curve. To avoid this, the number of grids should be fixed by explicitly specifying the following keyword:

scf.Ngrid      32 32 32

The numbers correspond to the number of grid along a-, b-, and c-axes, respectively. scf.Ngrid is used if both the keywords scf.energycutoff and scf.Ngrid are specified.



MD.EvsLC.flag    1 1 0      # along a, b, c-axes  
1: varied, 0: fixed

# Self-consistency: Simple charge mixing

The KS effective is constructed from  $\rho$ .

However,  $\rho$  is evaluated from eigenfaunctions of KS eq.

$$\hat{H}_{\text{KS}}\phi_i = \varepsilon_i\phi_i \quad \hat{H}_{\text{KS}} = -\frac{1}{2}\nabla^2 + v_{\text{eff}}$$

$$v_{\text{eff}} = v_{\text{ext}}(\mathbf{r}) + v_{\text{Hartree}}(\mathbf{r}) + \frac{\delta E_{\text{xc}}}{\delta \rho(\mathbf{r})}$$

$$\rho(\mathbf{r}) = \sum_i \phi_i^*(\mathbf{r})\phi_i(\mathbf{r})$$

## Simple charge mixing method

The next input density is constructed by a simple mixing of input and output densities.

$$\rho_{n+1}^{(\text{in})} = \alpha \rho_n^{(\text{in})} + (1 - \alpha) \rho_n^{(\text{out})},$$

It works well for large gap systems and small sized systems.

# Self-consistency: RMM-DIIS

## Idea:

Minimize the norm of a linear combination of previous residual vectors.

$$\bar{R}_{n+1} = \sum_{m=n-(p-1)}^n \alpha_m R_m,$$

$$R_n(\mathbf{q}) \equiv \tilde{n}_n^{(\text{out})}(\mathbf{q}) - \tilde{n}_n^{(\text{in})}(\mathbf{q}),$$

$$F = \langle \bar{R}_{n+1} | \bar{R}_{n+1} \rangle - \lambda \left( 1 - \sum_m^n a_m \right),$$

$$\langle R_m | R_{m'} \rangle \equiv \sum_{\mathbf{q}} \frac{R_m^*(\mathbf{q}) R_{m'}(\mathbf{q})}{w(\mathbf{q})},$$

$$= \sum_{m,m'} \alpha_m \alpha_{m'} \langle R_m | R_{m'} \rangle - \lambda \left( 1 - \sum_m^n a_m \right).$$

Kerker factor

$$w(\mathbf{q}) = \frac{|\mathbf{q}|^2}{|\mathbf{q}|^2 + q_0^2},$$

Minimization of F leads to

Long wave length components corresponding to small  $|\mathbf{q}|$  are taken into account.

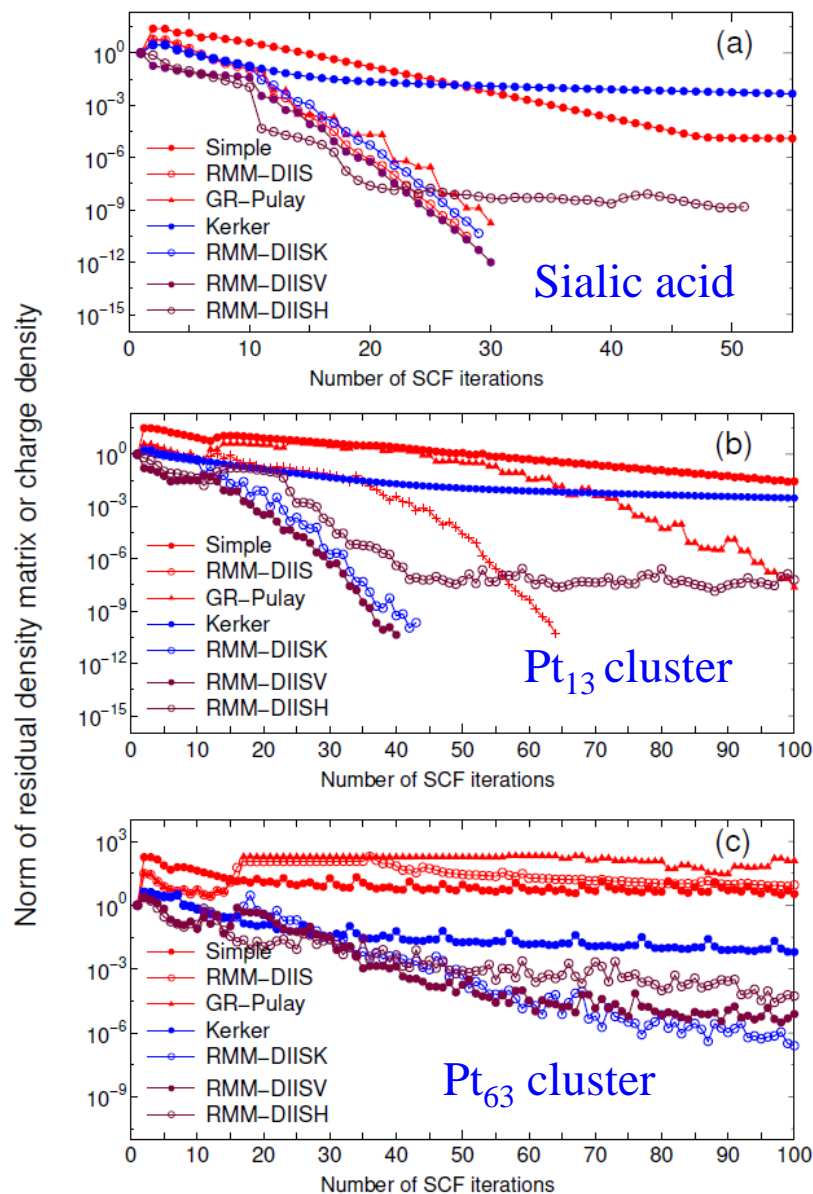
$$\frac{\partial F}{\partial \alpha_k} = 0 \quad \left( \begin{array}{cccc} \langle R_{n-(p-1)} | R_{n-(p-1)} \rangle & \cdots & \cdots & 1 \\ \cdots & \cdots & \cdots & 1 \\ \cdots & \cdots & \langle R_n | R_n \rangle & \cdots \\ 1 & 1 & \cdots & 0 \end{array} \right) \left( \begin{array}{c} \alpha_{n-(p-1)} \\ \alpha_{n-(p-1)+1} \\ \vdots \\ \frac{1}{2}\lambda \end{array} \right) = \left( \begin{array}{c} 0 \\ 0 \\ \vdots \\ 1 \end{array} \right).$$

Optimum input density might be given by

$$\rho_{n+1}^{(\text{in})} = \sum_{m=n-(p-1)}^n \alpha_m \rho_m^{(\text{in})} + \beta \sum_{m=n-(p-1)}^n \alpha_m R_m$$



# Mixing methods



## Available mixing methods:

Simple mixing (Simple)

Residual minimization method in the direct inversion iterative subspace (RMM-DIIS)

Guaranteed reduction Pulay method (GR-Pulay)

Kerker mixing (Kerker)

RMM-DIIS with Kerker metric (RMM-DIISK)

RMM-DIIS for Kohn-Sham potential (RMM-DIISV)

RMM-DIIS for Hamiltonian (RMM-DIISH)

Recommendation:

**RMM-DIISK or RMM-DIISV**

For DFT+U and constrained methods

**RMM-DIISH**

**See also the page 63 in the manual.**

# Database of PAO and VPS

[https://t-ozaki.issp.u-tokyo.ac.jp/vps\\_pao2019/](https://t-ozaki.issp.u-tokyo.ac.jp/vps_pao2019/)

## Database (2019) of optimized VPS and PAO

The database (2019) of fully relativistic pseudopotentials (VPS) and pseudo-atomic orbitals (PAO), generated by ADPACK, which could be an input data of program package, OpenMX. The data of elements with the underline are currently available. When you use these data, VPS and PAO, in the program package, OpenMX, then copy them to the directory, openmx\*/DFT\_DATA19/VPS/ and openmx\*/DFT\_DATA19/PAO/, respectively. The delta gauge of OpenMX with the database (2019) is found at [here](#).

<u>E</u>	Public release of optimized and well tested VPS and PAO so that users can easily start their calculations.																<u>He</u>
<u>Li</u>	<u>Be</u>											<u>B</u>	<u>C</u>	<u>N</u>	<u>O</u>	<u>F</u>	<u>Ne</u>
<u>Na</u>	<u>Mg</u>											<u>Al</u>	<u>Si</u>	<u>P</u>	<u>S</u>	<u>Cl</u>	<u>Ar</u>
<u>K</u>	<u>Ca</u>	<u>Sc</u>	<u>Ti</u>	<u>V</u>	<u>Cr</u>	<u>Mn</u>	<u>Fe</u>	<u>Co</u>	<u>Ni</u>	<u>Cu</u>	<u>Zn</u>	<u>Ga</u>	<u>Ge</u>	<u>As</u>	<u>Se</u>	<u>Br</u>	<u>Kr</u>
<u>Rb</u>	<u>Sr</u>	<u>Y</u>	<u>Zr</u>	<u>Nb</u>	<u>Mo</u>	<u>Tc</u>	<u>Ru</u>	<u>Rh</u>	<u>Pd</u>	<u>Ag</u>	<u>Cd</u>	<u>In</u>	<u>Sn</u>	<u>Sb</u>	<u>Te</u>	<u>I</u>	<u>Xe</u>
<u>Cs</u>	<u>Ba</u>	L	<u>Hf</u>	<u>Ta</u>	<u>W</u>	<u>Re</u>	<u>Os</u>	<u>Ir</u>	<u>Pt</u>	<u>Au</u>	<u>Hg</u>	<u>Tl</u>	<u>Pb</u>	<u>Bi</u>	<u>Po</u>	At	<u>Rn</u>
Fr	Ra	A															
	L	<u>La</u>	<u>Ce</u>	<u>Pr</u>	<u>Nd</u>	<u>Pm</u>	<u>Sm</u>	Eu	Gd	Tb	<u>Dy</u>	<u>Ho</u>	Er	Tm	Yb	<u>Lu</u>	
	A	Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr	

# Specification of PAO and VPS

PAO and VPS are specified by the following keyword:

```
<Definition.of.Atomic.Species  
  O    O7.0-s2p2d1      O_PBE19  
  H    H7.0-s2p1       H_PBE19  
>Definition.of.Atomic.Species
```

- O7.0 means O7.0.pao.
- -s2p2d1 means 2, 2, and 1 radial functions are allocated to s-, p-, and d-orbitals.
- In this case, for oxygen atom,  $2 \times 1 + 2 \times 3 + 1 \times 5 = 13$  basis functions are allocated.
- O\_PBE19 means O\_PBE19.vps.

The path for O7.0.pao and O\_PBE19.vps is specified by

```
DATA.PATH    /home/soft/openmx3.9/DFT_DATA19
```

Default value is ‘../DFT\_DATA19’.

# How to choose basis functions: H<sub>2</sub>O case

By clicking H7.0.pao and O7.0.pao in the database(2019), you may find the following

[https://t-ozaki.issp.u-tokyo.ac.jp/vps\\_pao2019/H/index.html](https://t-ozaki.issp.u-tokyo.ac.jp/vps_pao2019/H/index.html)

\*\*\*\*\*  
Eigen values(Hartree) of pseudo atomic orbitals  
\*\*\*\*\*

Eigenvalues H7.0.pao

Lmax= 3 Mul=15

mu	0	0	-0.23595211038442
mu	0	1	0.14109389991827
mu	0	2	0.61751730037441
mu	0	3	1.31890671598573
mu	0	4	2.24052765608302
mu	0	5	3.37954791544661
mu	0	6	4.73488369825610
mu	0	7	6.30608874470710
mu	0	8	8.09282718517299
mu	0	9	10.09464035732420
mu	0	10	12.31085267019158
mu	0	11	14.74057314485273
mu	0	12	17.38277845742691
mu	0	13	20.23645090753857
mu	0	14	23.30073926597344
mu	1	0	0.10914684890465
mu	1	1	0.47776040452236
mu	1	2	1.06988680483686
mu	1	3	1.88261331124981
mu	1	4	2.91175885838084
mu	1	5	4.15601184789448
mu	1	6	5.61454131060210
mu	1	7	7.28681796296307
mu	1	8	9.17254361476158
mu	1	9	11.27156766390586
mu	1	10	13.58381334569800
mu	1	11	16.10921637499960
mu	1	12	18.84767560575107
mu	1	13	21.79902110024685
mu	1	14	24.96300480798286
mu	2	0	0.27851528500170
mu	2	1	0.76970400958463
mu	2	2	1.47576814497054
mu	2	3	2.39881523735167
mu	2	4	3.53713246447754

3  
6

5

[https://t-ozaki.issp.u-tokyo.ac.jp/vps\\_pao2019/O/index.html](https://t-ozaki.issp.u-tokyo.ac.jp/vps_pao2019/O/index.html)

\*\*\*\*\*  
Eigen values(Hartree) of pseudo atomic orbitals  
\*\*\*\*\*

Eigenvalues O7.0.pao

Lmax= 3 Mul=15

mu	0	0	-0.87913976280231
mu	0	1	0.06809061901229
mu	0	2	0.52709275865941
mu	0	3	1.24995140722317
mu	0	4	2.21829552402723
mu	0	5	3.41945468859267
mu	0	6	4.84509607059749
mu	0	7	6.48825090142865
mu	0	8	8.34207134316001
mu	0	9	10.39973244132192
mu	0	10	12.65513764955926
mu	0	11	15.10428688136984
mu	0	12	17.74677530362947
mu	0	13	20.58633940582683
mu	0	14	23.62957122674031
mu	1	0	-0.33075182895384
mu	1	1	0.16376499567753
mu	1	2	0.64129274864838
mu	1	3	1.35995471521821
mu	1	4	2.31377697411480
mu	1	5	3.50052618379026
mu	1	6	4.91841590421346
mu	1	7	6.56499793348396
mu	1	8	8.43634105410091
mu	1	9	10.52733540479795
mu	1	10	12.83276989780839
mu	1	11	15.34880171573570
mu	1	12	18.07414632417004
mu	1	13	21.01013110934429
mu	1	14	24.15933607166566
mu	2	0	0.26162948257116
mu	2	1	0.70705247436937
mu	2	2	1.34714706672243
mu	2	3	2.19799459356269
mu	2	4	3.26511989658328

1  
4

2  
7

8

Choosing states with lower eigenvalues leads to H7.0-s2p1 and O7.0-s2p2d1.

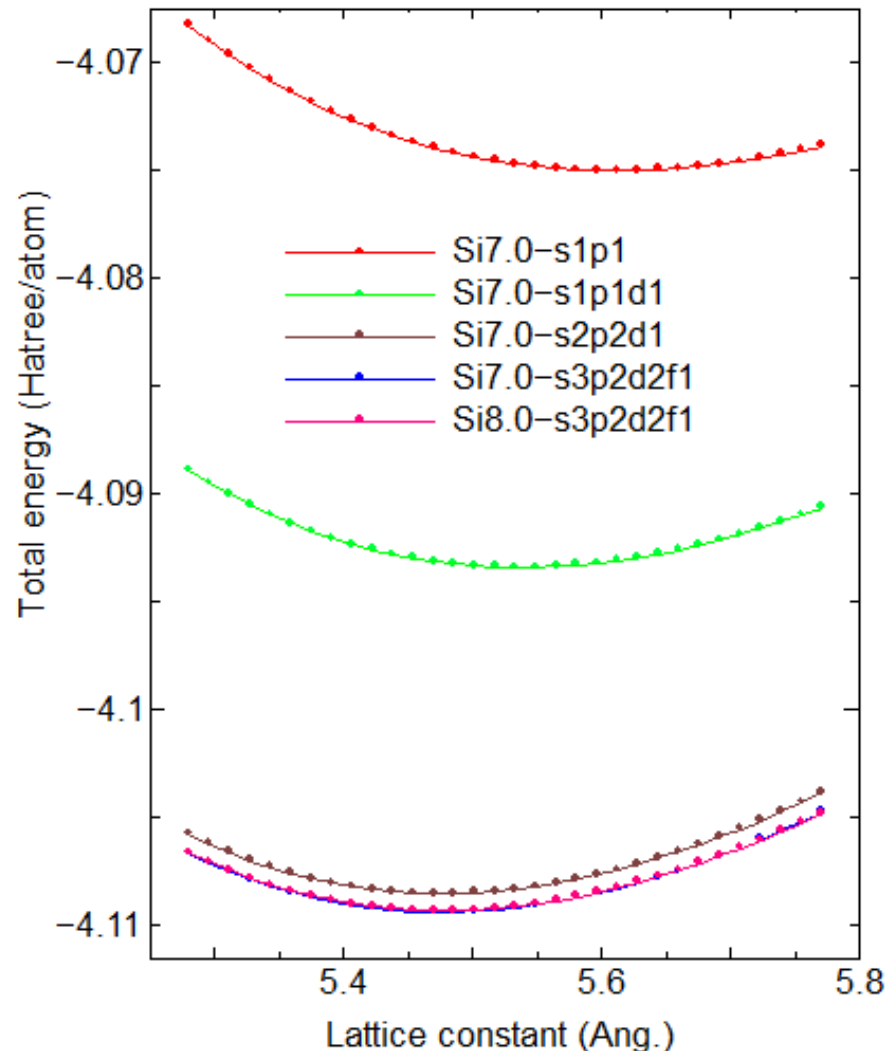
# How to choose basis functions: Si case(1)

Eigenvalues  
Lmax= 3 Mul=15

Si7.0.pao

mu	0	0	-0.59320968623145	1
mu	0	1	0.01654247265872	4
mu	0	2	0.57915688461708	8
mu	0	3	1.39915528592998	
mu	0	4	2.43106703909053	
mu	0	5	3.63498884739274	
mu	0	6	4.99885012002783	
mu	0	7	6.55485759476897	
mu	0	8	8.33541526908920	
mu	0	9	10.34055461939939	
mu	0	10	12.55818051232040	
mu	0	11	14.98369777887096	
mu	0	12	17.62014479571136	
mu	0	13	20.47011746372617	
mu	0	14	23.53245309073866	
mu	1	0	-0.31460013133101	2
mu	1	1	0.12937640798489	5
mu	1	2	0.71637380083701	
mu	1	3	1.54488697995006	
mu	1	4	2.59211686526084	
mu	1	5	3.84687324239366	
mu	1	6	5.31246180826158	
mu	1	7	6.99509799702661	
mu	1	8	8.89454075291723	
mu	1	9	11.00602102880752	
mu	1	10	13.32692005826443	
mu	1	11	15.85857455673626	
mu	1	12	18.60272353142246	
mu	1	13	21.55894843635971	
mu	1	14	24.72602323954447	
mu	2	0	0.01886411574821	3
mu	2	1	0.35893514996065	7
mu	2	2	0.94629918754692	
mu	2	3	1.76201765644983	
mu	2	4	2.81418723624388	
mu	2	5	4.10656012645961	
mu	2	6	5.63661971875011	
mu	2	7	7.39522693820483	
mu	2	8	9.37222098331867	
mu	2	9	11.56227937764802	
mu	2	10	13.96620568880690	
mu	2	11	16.58651812641581	
mu	2	12	19.42296574188659	
mu	2	13	22.47308081363278	
mu	2	14	25.73538272482773	
mu	3	0	0.28356769151846	6
mu	3	1	0.79082836114569	
mu	3	2	1.52992065308726	
mu	3	3	2.52537261496132	

Orbitals with lower eigenvalues in Si7.0.pao are taken into account step by step as the quality of basis set is improved.



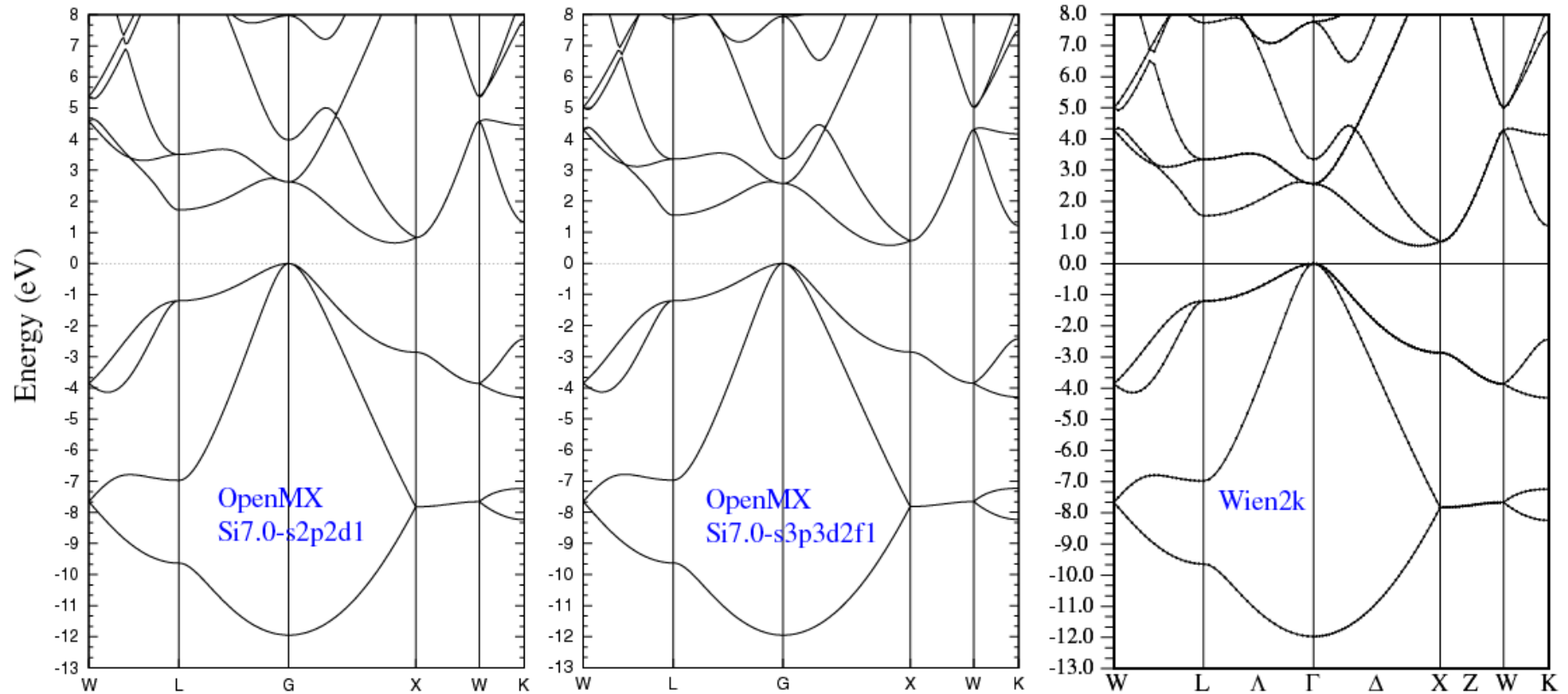
Si7.0-s2p2d1 is enough to discuss structural properties.

By comparing Si7.0-s3p2d2f1 with Si8.0-s3p2d2f1, it turns out that convergence is achieved at the cutoff of 7.0(a.u.).

# How to choose basis functions: Si case(2)

With respect to band structure, one can confirm that Si7.0-s2p2d1 provides a nearly convergent result.

[https://t-ozaki.issp.u-tokyo.ac.jp/vps\\_pao2019/Si/index.html](https://t-ozaki.issp.u-tokyo.ac.jp/vps_pao2019/Si/index.html)



While the convergent result is achieved by use of Si7.0-s3p2d2f1(Si7.0-s3p3d2f1), Si7.0-s2p2d1 is a balanced basis functions compromising accuracy and efficiency to perform a vast range of materials exploration.

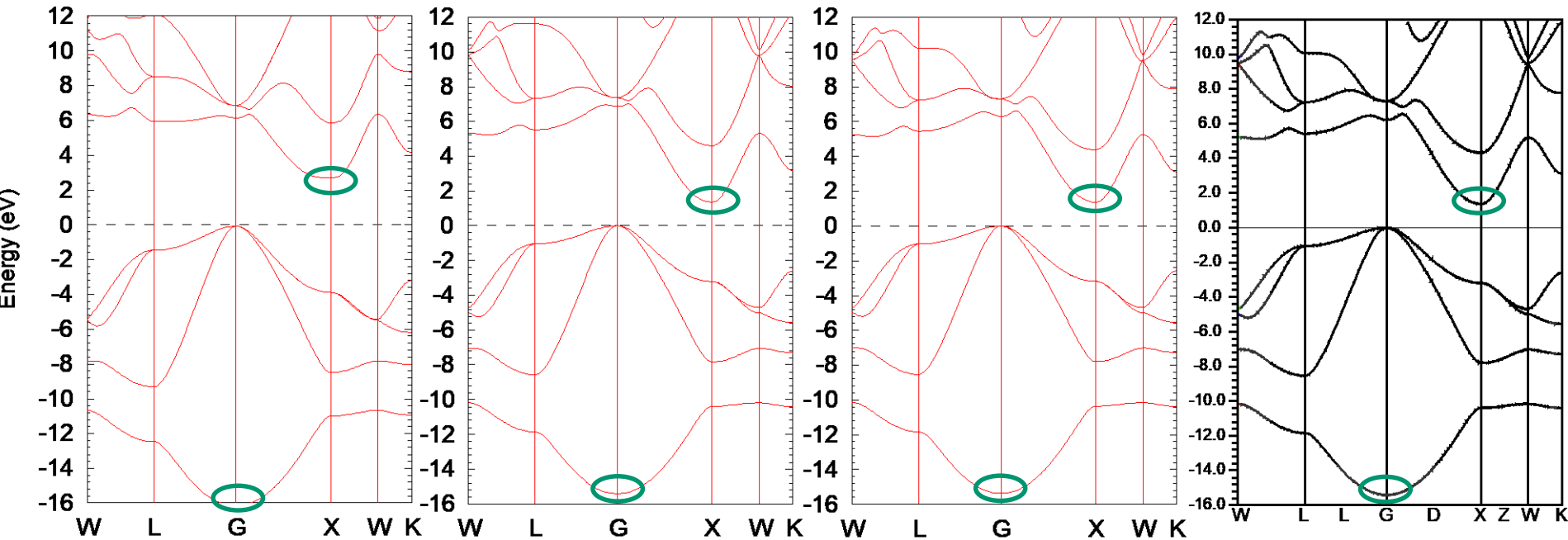
# Floating states in 3C-SiC

C6.0-s2p2  
Si8.0-s2p2

C6.0-s2p2d1  
Si8.0-s2p2d1

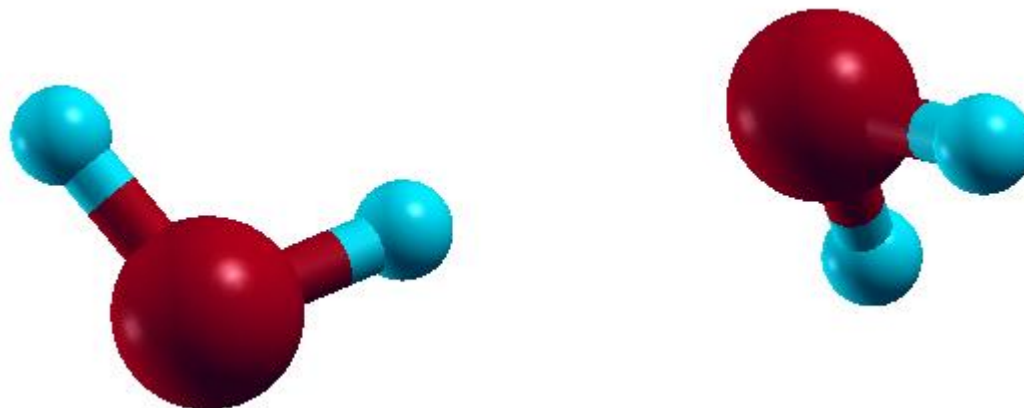
C6.0-s3p3d2f1  
Si8.0-s3p3d2f1

Wien2k (FLAPW)



- Inclusion of polarization orbitals is important to reproduce band structures.
- The band structure up to 5 eV is reproduced by s2p2d1.

# Basis set superposition Error (BSSE)



	Equilibrium O-O distance (Ang.)	Dipole moment (Debye)	Binding energy (kcal/mol)	Binding energy (counterpoise corrected) (kcal/mol)
O7.0-s2p2d1, H7.0-s2p1	2.899	2.54	5.57	5.21
O7.0-s3p3d2, H7.0-s3p2	2.897	2.45	5.48	5.48
Other calc.	2.893 <sup>a</sup>		5.15 <sup>a</sup>	
Expt.	2.98 <sup>b</sup>	2.60 <sup>b</sup>	5.44 <sup>b</sup>	

[https://t-ozaki.issp.u-tokyo.ac.jp/vps\\_pao2019/O/index.html](https://t-ozaki.issp.u-tokyo.ac.jp/vps_pao2019/O/index.html)

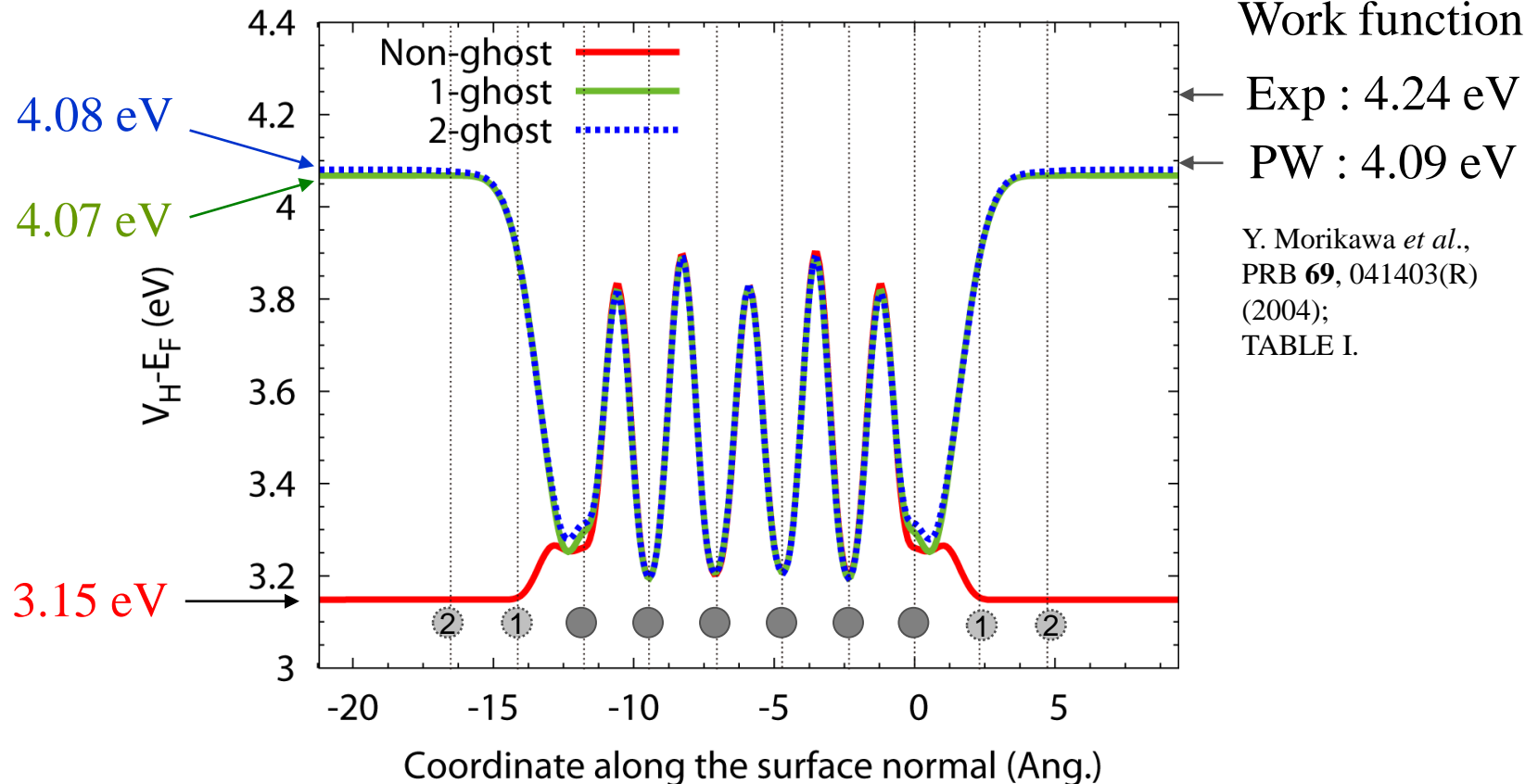
A series of benchmark calculations implies that BSSE is  
 $\sim 0.5$  kcal/mol for molecular systems.



# Work functions

## fcc Al (111) surface

By allocating empty atoms in vacuum near the surface, one can calculate work functions accurately.



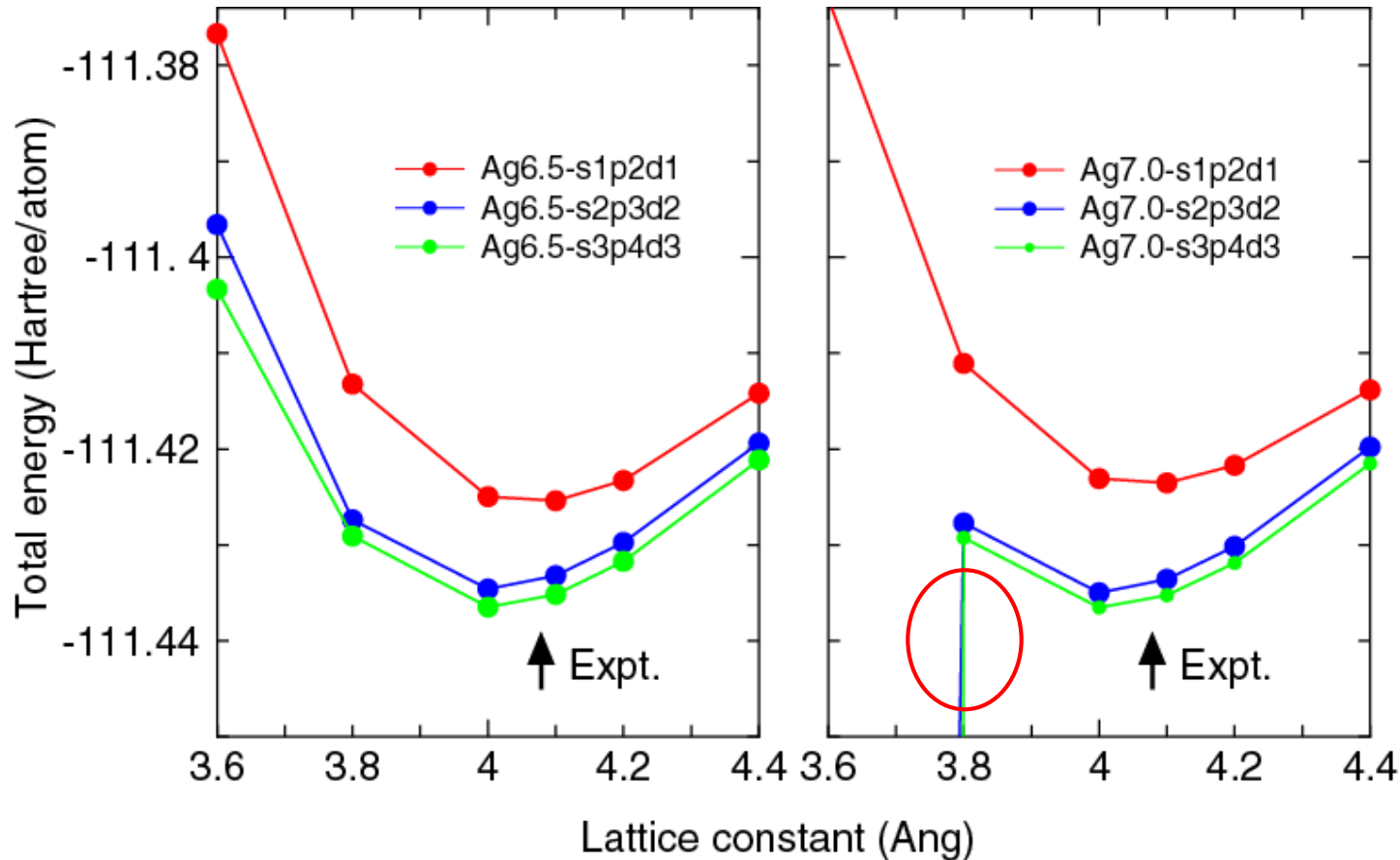
By Jippo and Ohfuti (Fujitsu)

Al(111) fcc 1x1 6L, GGA-PBE, Fix to the equilibrium lattice parameter  
Al6.0-slp2d1, Al\_PBE (Database ver. 2006)  
E6.0-slp2d1, E (Database ver. 2006)

See the page 207 in the manual.

# Overcompleteness of basis functions

Total energy of fcc-Ag



A numerical instability, called “overcompleteness”, tends to appear if a lot of basis functions are used for dense structures such as fcc, hcp, and bcc.

# Cause of overcompleteness

Let's take a  $H_2$  molecule as a simple example, and consider  $d \rightarrow 0$  where  $d$  is the interatomic distance.

$$Hc = \varepsilon Sc$$

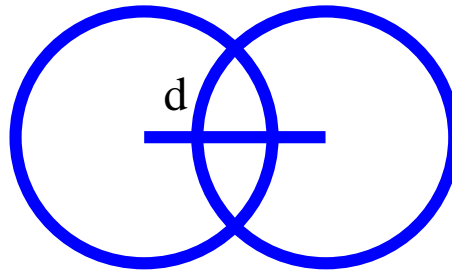
$$H = \begin{pmatrix} e & h \\ h & e \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & s \\ s & 1 \end{pmatrix}$$

$$S = V \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} V^\dagger = V \lambda V^\dagger$$

$$\lambda_1 = 1 - s \quad v_1 = (1, -1)$$

$$\lambda_2 = 1 + s \quad v_2 = (1, 1)$$



$$Hc = \varepsilon Sc$$

$$Hc = \varepsilon V \lambda V^\dagger c$$

$$\lambda^{-1/2} V^\dagger H V \lambda^{-1/2} \lambda^{1/2} V^\dagger c = \varepsilon \lambda^{1/2} V^\dagger c$$

$$H' = \lambda^{-1/2} V^\dagger H V \lambda^{-1/2}$$

$$b = V^\dagger c$$

$$H'b = \varepsilon b$$

A round-off error arises when  $H'$  is calculated, and the error is magnified by  $1/\sqrt{\lambda_1}$ .

# Recommendation of choices for PAO

VPS	Valence electrons	Quick	Standard	Precise
E	0.0	Kr10.0-s1p1	Kr10.0-s2p1d1	Kr10.0-s2p2d1f1
H_PBE19	1.0	H5.0-s2	H6.0-s2p1	H7.0-s2p2d1
He_PBE19	2.0	He8.0-s1p1	He8.0-s2p1	He10.0-s2p2d1
Li_PBE19	3.0	Li8.0-s3p1	Li8.0-s3p2	Li8.0-s3p2d1
Be_PBE19	2.0	Be7.0-s2p1	Be7.0-s2p2	Be7.0-s3p2d1
B_PBE19	3.0	B7.0-s2p2	B7.0-s2p2d1	B7.0-s3p2d2
C_PBE19	4.0	C6.0-s2p2	C6.0-s2p2d1	C6.0-s3p2d2
N_PBE19	5.0	N6.0-s2p2	N6.0-s2p2d1	N6.0-s3p2d2
O_PBE19	6.0	O6.0-s2p2	O6.0-s2p2d1	O6.0-s3p2d2
F_PBE19	7.0	F6.0-s2p2	F6.0-s2p2d1	F6.0-s3p3d2f1
Ne_PBE19	8.0	Ne9.0-s2p2	Ne9.0-s2p2d1	Ne9.0-s3p2d2
Na_PBE19	9.0	Na9.0-s3p2	Na9.0-s3p2d1	Na9.0-s3p2d2
Mg_PBE19	8.0	Mg9.0-s2p2	Mg9.0-s3p2d1	Mg9.0-s3p2d2
Al_PBE19	3.0	Al7.0-s2p1d1	Al7.0-s2p2d1	Al7.0-s3p2d2
Si_PBE19	4.0	Si7.0-s2p1d1	Si7.0-s2p2d1	Si7.0-s3p3d2
P_PBE19	5.0	P7.0-s2p2d1	P7.0-s2p2d1f1	P7.0-s3p2d2f1
S_PBE19	6.0	S7.0-s2p2d1	S7.0-s2p2d1f1	S7.0-s3p2d2f1
Cl_PBE19	7.0	Cl7.0-s2p2d1	Cl7.0-s2p2d1f1	Cl7.0-s3p2d2f1

See also the pages 55 and 56 in the manual.

# Restarting of calculations

- After finishing your first calculation or achieving the self consistency, you may want to continue the calculation or to calculate density of states, band dispersion, molecular orbitals, and etc. using the self consistent charge in order to save the computational time. To do this, a keyword 'scf.restart' is available.

`scf.restart`            `on`            `# on|off,default=off`

- If the first trial for geometry optimization does not reach a convergent result or molecular dynamics simulation is terminated due to a wall time, one can restart the geometry optimization using an input file 'System.Name.dat#' which is generated at every step for the restart calculation with the final structure.

See also the page 67 in the manual.

# Output of large-sized files in binary mode

Large-scale calculations produce large-sized files in text mode such as cube files. The IO access to output such files can be very time consuming in machines of which IO access is not fast. In such a case, it is better to output those large-sized files in binary mode. The procedure is supported by the following keyword:

`OutData.bin.flag on # default=off, on|off`

Then, all large-sized files will be output in binary mode. The default is 'off'. The output binary files are converted using a small code 'bin2txt.c' stored in the directory 'source' which can be compiled as

```
gcc bin2txt.c -lm -o bin2txt
```

As a post processing, you will be able to convert as

```
./bin2txt *.bin
```

The functionality will be useful for machines of which IO access is not fast.

See also the page 320 in the manual.

# Large-scale calculations

The following is a result of 'runtestL2' performed using 640 MPI processes and 1 OpenMP threads on a Xeon cluster.

```
$ mpirun -np 640 openmx -runtestL2 -nt 1
```

1	large2_example/C1000.dat	Elapsed time(s)= 777.60	diff Utot= 0.000000007341	diff Force= 0.000000008795
2	large2_example/Fe1000.dat	Elapsed time(s)= 8181.70	diff Utot= 0.000000002241	diff Force= 0.000000011061
3	large2_example/GRA1024.dat	Elapsed time(s)= 927.20	diff Utot= 0.000000012903	diff Force= 0.000000004981
4	large2_example/Ih-Ice1200.dat	Elapsed time(s)= 445.88	diff Utot= 0.000000000216	diff Force= 0.000000001451
5	large2_example/Pt500.dat	Elapsed time(s)= 2629.20	diff Utot= 0.000000015832	diff Force= 0.000000001879
6	large2_example/R-TiO2-1050.dat	Elapsed time(s)= 844.58	diff Utot= 0.000000002263	diff Force= 0.000000001108
7	large2_example/Si1000.dat	Elapsed time(s)= 658.53	diff Utot= 0.000000000404	diff Force= 0.000000000908

Total elapsed time (s)            14464.69

The elapsed time implies that geometry optimization for systems consisting of 1000 atoms is possible if several hundreds processor cores are available.

See also the page 114 in the manual.

# Visualization

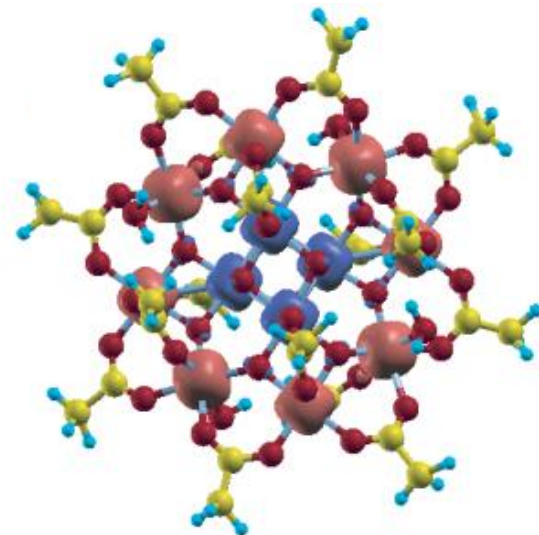
- Cube files such as \*.tden.cube, \*.sden.cube, \*.dden.cube can be visualized by many software such as

XCrySDen  
VESTA

- Also \*.md file is stored in xyz format which can be visualized by XCrySDen and OpenMX Viewer.

On phi, XCrSDen is available.

- Data on DOS and band dispersion can be visualized by gnuplot.





# On the manual

The PDF file is available at

[http://www.openmx-square.org/openmx\\_man3.9/openmx3.9.pdf](http://www.openmx-square.org/openmx_man3.9/openmx3.9.pdf)

The manual is self-contained, the most of calculations explained in the manual are traceable by using the input file stored in the directory ‘work’.

Please try to perform those calculations one by one depending on your interests.